1. Let \((X, R)\) be a range space where \(|X| = n\) and \(|R| = m\) and \(m > n\). Show that for any \(0 < \varepsilon < 1\), there is an \(\varepsilon\)-net of \(X\) whose size is \(O(\frac{\log m}{\varepsilon})\). Use the probabilistic method and the Chernoff bound.

2. Let \((A, R)\) be a range space. Let \(Z \subseteq Y \subseteq X\) be subsets of \(A\), and suppose that for \(0 < \varepsilon, \delta < 1\), \(Y\) is an \(\varepsilon\)-net of \(X\) and \(Z\) is a \(\delta\)-net of \(Y\). Show that \(Z\) is an \((\varepsilon + \delta)\)-net of \(X\).

3. Let \((A, R)\) be a range space. Let \(X_1\) and \(X_2\) be two disjoint subsets of \(A\) such that \(|X_1| = |X_2|\), and let \(Y_1 \subseteq X_1\) and \(Y_2 \subseteq X_2\) be subsets such that \(|Y_1| = |Y_2|\). For some \(0 < \varepsilon < 1\), suppose that \(Y_1\) is an \(\varepsilon\)-net of \(X_1\) and \(Y_2\) is an \(\varepsilon\)-net of \(X_2\). Show that \(Y_1 \cup Y_2\) is an \(\varepsilon\)-net of \(X_1 \cup X_2\).

4. Let \((X, R)\) be an infinite range space. Suppose that we are told that for any finite subset \(A \subseteq X\), \(\Pi_R(A) = \{s \mid s = s \cap r \text{ for some } r \in R\}\) has size \(O(|A|^d)\), where \(d\) is a constant independent of \(|A|\). Argue that \((X, R)\) has finite VC-dimension.

5. Conclude that the range space \((X, R)\) where \(X = \mathbb{R}^2\) and \(R\) is the set of all half-planes has finite VC-dimension.

6. Use the previous two problems to show that the range space \((X, R)\) where \(X = \mathbb{R}^2\) and \(R\) is the set of all triangles in the plane has finite VC-dimension.