22C : 153 **Design and Analysis of Algorithms**
Spring 2002
Homework 5
Due on May 2

1. In class, we wrote a maximization linear program corresponding to the shortest path problem, and we showed that the value of the optimal solution to this linear program is the length of the shortest path. Describe a scheme to extract the shortest path itself from the solution of the linear program. (Hint: Look carefully at the proof we did in class.)

2. Consider the following "primal" linear program:

$$\text{Minimize } \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to } \sum_{j=1}^{n} a_{ij} x_j \geq b_i \text{ for each } 1 \leq i \leq m$$

$$x_j \geq 0 \text{ for each } \ 1 \leq j \leq n.$$

Note that the only difference from the "slack" form that we considered in class is that the m equalities are replaced by inequalities. The "dual" linear program is:

$$\text{Maximize } \sum_{i=1}^{m} b_i y_i$$

$$\text{Subject to } \sum_{i=1}^{m} a_{ij} y_i \leq c_j \text{ for each } 1 \leq j \leq n$$

$$y_i \geq 0 \text{ for each } 1 \leq i \leq m.$$

Note that the only difference from the dual program we wrote in class is the non-negativity constraint on the dual variables.

(a) Prove the weak duality theorem for these programs. That is, show that if $\hat{x} = (\hat{x_1}, \ldots, \hat{x_n})$ is a feasible solution for the primal and $\hat{y} = (\hat{y_1}, \ldots, \hat{y_m})$ is a feasible solution for the dual, then

$$\sum_{i=1}^{m} b_i \hat{y_i} \leq \sum_{j=1}^{n} c_j \hat{x_j}.$$

(b) Write down, with proof, the complementary slackness conditions for a feasible solution $\hat{x}$ for the primal and a feasible solution $\hat{y}$ for the dual to be optimal.

Both these problems can be solved in a manner analogous to what we did in class.

3. Exercise 34.2-1

4. Exercise 34.2-5

5. Exercise 34.3-2

6. Exercise 34.3-8