

Limits of Computation (CS:4340:0001 or 22C:131:001)
Homework 5

The homework is due in class on Tuesday, December 1. If you can't make it to class, drop it in my mailbox in the MacLean Hall mailroom.

1. Show that the following language SPACE TMSAT is PSPACE-complete. (2.5 points).

$$\text{SPACE TMSAT} = \{\langle \alpha, w, 1^n \rangle : M_\alpha \text{ is a TM and it accepts } w \text{ in space } n.\}$$

Note that $\alpha, w \in \{0, 1\}^*$. The language SPACE TMSAT is introduced in page 83 of the text.

2. Describe a polynomial time algorithm that given a quantified boolean formula (QBF) ψ outputs an equivalent QBF ψ' in prenex normal form. Illustrate your algorithm using an example or two. (The algorithm is briefly given in Section 4.2, in the penultimate paragraph on page 83.) Formally argue the equivalence for at least one of the 3 cases (and, or, and not). (2.5 points)

ψ and ψ' are said to be equivalent if (a) the set of free variables is the same in both formulae, and (b) under any assignment b to the free variables, $\psi|_b$ and $\psi'|_b$ evaluate to the same quantity. If both ψ and ψ' have no free variables, they are equivalent if they evaluate to the same quantity.

3. Let L be a language that is not the empty set or $\{0, 1\}^*$. Argue that for any language L' in the class \mathbf{NL} , $L' \leq_P L$. The class \mathbf{NL} stands for $\mathbf{NSPACE}(\log n)$. (2.5 points)

Hint: What is the relationship between \mathbf{NL} and \mathbf{P} ?

4. Use the time hierarchy theorem to conclude that there is a language L such that $L \in \mathbf{EXP}$ and $L \notin \mathbf{P}$.