Limits of Computation (CS:4340:0001 or 22C:131:001)
Homework 1

The homework is due in class on Tuesday, September 8th. If you can't make it to class, drop it in my mailbox in the MacLean Hall mailroom.

1. Let Pattern : $\{0,1\}^* \to \{0,1\}$ be the function defined as follows: Pattern$(x) = 1$ if 101 occurs as a contiguous substring in x, and 0 otherwise. For example, Pattern$(11010) = 1$, and Pattern$(11001) = 0$. Describe a Turing Machine that computes the function Pattern. What is its running time, in big-Oh notation, as a function of the input size? For this question, a full TM description is expected. (2.5 points)

2. Let $f : \{0,1\}^* \to \{0,1\}$ be defined as follows: $f(x) = 1$ if $x = 0^n1^n0^n$ for some integer $n \geq 0$, and $f(x) = 0$ otherwise. For example, $f(001100) = 1$, $f(0110) = 0$, and $f(1100) = 0$. Describe a Turing Machine that computes the function f. What is its running time, in big-Oh notation, as a function of the input size? You don't need to give the full TM description, but give the kind of detail we gave in proving Claim 1.5 in class – explain the states of the TM, and the pattern in which they are traversed. (2.5 points)

3. Describe a TM that on input $x \in \{0,1\}^*$ outputs the binary representation of $|x|$. The running time should be $O(n)$ where n is $|x|$, the length of x. In this and the next question, describe the workings of the TM at a higher level, but you should be confident about being able to translate this description to a full transition function if an alien asks you to. (2.5 points)

4. Describe a TM that on input $x \in \{0,1\}^*$ outputs the binary representation of $|x|^2$. The running time should be $O(n^2)$ where $n = |x|$. (2.5 points)

For each problem, you may use as many alphabet symbols or tapes as convenient. When giving details of the transition function, complement them with high level comments; this will aid the reader's understanding.