This is closed book exam. You have an hour and fifteen minutes.

1. Arrange the following running times in a sequence in such a way that any element in the sequence is big-O of the succeeding element.

   (a) \( n \log n \)
   (b) \( n^2 \)
   (c) \( n \log^3 n \)
   (d) \( 1.7^n \)
   (e) \( n^{2.5} \)

2. Show the sequence of the set of engaged pairs in an execution of the stable matching algorithm involving the three men \( m_1, m_2, m_3 \) and the three women \( w_1, w_2, w_3 \) with the following preferences:
   - \( m_1 : w_1 > w_2 > w_3 \)
   - \( m_2 : w_1 > w_3 > w_2 \)
   - \( m_3 : w_3 > w_1 > w_2 \)
   - \( w_1 : m_2 > m_3 > m_1 \)
   - \( w_2 : m_1 > m_2 > m_3 \)
   - \( w_3 : m_1 > m_3 > m_2 \)

3. Suppose that we call the recursive \( O(n \log^3 n) \) algorithm for multiplying two polynomials whose coefficients are represented by the following arrays:

   - A: 1 7 2 4 6 3 5 2
   - B: 3 0 7 6 3 7 6 9

   What are the coefficient arrays that are passed to each of the three recursive calls made by the algorithm?

4. Consider the \( O(n \log^2 n) \) algorithm we discussed in class (or the \( O(n \log n) \) algorithm in the textbook) for finding the closest pair in a given set of \( n \) points in the plane. Write down a recurrence for upper bounding the number of pairs of points whose distance is computed by the algorithm. What does this recurrence solve to? Derive as tight a bound as possible in the big-O notation.

5. The following algorithm takes as input an array \( A \) of \( n \) integers and a target integer \( t \), and checks if there are two distinct elements in the array that add up to \( t \).
flag := false
For i from 1 to n-1 do
    For j from i+1 to n do
        if (A[i] + A[j] == t) then flag := true
    endfor
endfor
Return flag

(a) Give an asymptotically tight bound on the worst case running time of the algorithm as a function of n.

(b) Describe a new algorithm for the same problem with an asymptotically faster running time. (This is a design question, you should consider attempting it after attempting the other questions.)