1. Problem 15-2 (20 points)

2. Problem 15-4 (15 points)

3. Given a set of $n$ distinct positive integers $S = \{s_1, \ldots, s_n\}$, and a target integer $t$, determine if there is a subset $S' \subseteq S$ of $S$ such that the elements in $S'$ add up to $t$. For example within $S = \{1, 2, 5, 9, 10\}$ there is a subset which adds up to $t = 22$ but not $t = 23$. We want to develop a dynamic programming solution to this problem. (15 points.)

We say that a set $Q$ of positive integers *achieves* an integer $w$ if there is a subset $Q' \subseteq Q$ such that the elements of $Q'$ add up to $w$. By convention, we will assume that any set $Q$ achieves 0. Note that we want to determine if the set $S$ achieves $t$. For $1 \leq i \leq n$, let $S_i = \{s_1, \ldots, s_i\}$ be the set obtained by taking the first $i$ elements of $S$. Note that $S = S_n$. Let $S_0 = \emptyset$.

(a) Prove that for $1 \leq i \leq n$ and any $w > 0$, $S_i$ achieves $w$ if and only if either $S_{i-1}$ achieves $w$ or $S_{i-1}$ achieves $w - s_i$.

(b) Use this relation to give a dynamic programming algorithm that as a by-product determines if $S$ achieves $t$. The algorithm should run in $O(nt)$ time.