Instructor: W. A. Kirk

Name

Instructions. You may use calculators for this exam. (This policy is subject to change.) Guess if you must. Good Luck!

1. (20 points) The graph of \( g \) is given below.

(a) State the value of \( g(2) \).

(b) Why is \( g \) one-to-one?

(c) Estimate the value of \( g^{-1}(2) \).

(d) Estimate the domain of \( g^{-1} \).

(e) Sketch the graph of \( g^{-1} \) (on the same axes). (You should probably use a pencil!)

2. (5 points) Find a piecewise defined expression for the function whose graph consists of the line segment from the point \((-2, 2)\) to the point \((-1, 0)\) together with the top half of the circle with center at the origin and radius 1.
3. (20 points) Let $f(x) = \ln x$ and $g(x) = x^2 - 9$.

(a) Write an expression for the function $f \circ g$

(b) What is the domain of $f \circ g$?

(c) Write an expression for the function $f \circ f$.

(d) What is the domain of $f \circ f$?
4. (5 points) Find the inverse of the function \( f(x) = \frac{x + 1}{2x + 1} \).

5. (10 points) Solve for \( x \).

(a) \( 2x - 1 = e^{\ln x^2} \).

(b) \( 7^{x+4} = e^{17x} \). [Hint: \( 7 = e^{\ln 7} \).]
6. (22 points) Let

\[ f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leq x < 3 \\ (x - 3)^2 & \text{if } x > 3 \end{cases} \]

(a) Evaluate each limit if it exists. (It might help to sketch a graph of \( f \).)

(i) \( \lim_{x \to 0^+} f(x) \)

(ii) \( \lim_{x \to 0^-} f(x) \)

(iii) \( \lim_{x \to 0} f(x) \)

(iv) \( \lim_{x \to 3^-} f(x) \)

(v) \( \lim_{x \to 3^+} f(x) \)

(vi) \( \lim_{x \to 3} f(x) \)

(b) Where if \( f \) discontinuous?
7. (18 points) Determine the following limits (if they exist) [Note: You need do only THREE of the four problems. Clearly indicate which problem you wish to omit.]

(a) \( \lim_{x \to 1} \frac{1 - \sqrt{x}}{1 - x} \).

(b) \( \lim_{x \to 1} \arccos \frac{1 - \sqrt{x}}{1 - x} \).

(c) \( \lim_{x \to \infty} \frac{1 - \sqrt{2x^2 + 1}}{1 + \sqrt{x}} \).

(d) \( \lim_{x \to 0} \frac{1}{1 - 2^{1/x}} \).
8. Sketch a graph of an example of a function that satisfies all of the following conditions.

- \( \lim_{x \to 0^+} f(x) = -2 \),
- \( \lim_{x \to 0^-} f(x) = 1 \),
- \( \lim_{x \to -2^-} f(x) = \infty \),
- \( \lim_{x \to -2^+} f(x) = -\infty \),
- \( \lim_{x \to -\infty} f(x) = 3 \),
- \( \lim_{x \to -\infty} f(x) = 4 \),
- \( f(0) = -1 \).