Instructions. Show all your work where appropriate. You may NOT use calculators.

1. $\int xf'(x) \, dx =$

   (a) $f(x) - \int f(x) \, dx$

   (b) $xf(x) - \int f(x) \, dx$

   (c) $xf(x) - f(x)$

   (d) $f(x) - \int xf(x) \, dx$

   (e) $f(x)$

2. If $x = \tan \theta$ then $\sin 2\theta =$

   (a) $\frac{2}{1 + x^2}$

   (b) $\frac{1}{1 + x^2}$

   (c) $\frac{1}{\sqrt{1 + x^2}}$

   (d) $\frac{x}{\sqrt{1 + x^2}}$

   (e) $\frac{2x}{1 + x^2}$
3. (a) What is the partial fraction decomposition of \( \frac{1}{u(u-1)} \)?

(b) Evaluate \( \int \frac{du}{u(u-1)} \)

(c) Make the substitution \( u = 1 + e^x \) and transform \( \int \frac{dx}{1 + e^x} \) into an integral of the form \( \int f(u) \, du \). What is \( f(u) \)?

(d) Use the preceding answers to evaluate \( \int \frac{dx}{1 + e^x} \).
4. Evaluate \( \int \frac{x^2 + 2}{x + 2} \, dx. \)

5. Evaluate \( \int_{0}^{2} \frac{4}{(4 + x^2)^{3/2}} \, dx. \) [Make the substitution \( x = 2 \tan \theta. \)]
6. (a) Set up an integral that will give the length of \( y = \cos x \) on the interval \([0, 2\pi]\).

(b) Use left endpoint approximation with \( n = 4 \) to give an estimate of this length. [Simply your answer but leave it in ‘uncalculated’ form.]
7. Determine (a) the area, and (b) the centroid of the region bounded by $x = 5 - y^2$ and the $y$-axis. [See the figure below. You need not evaluate the definite integrals involved.]
8. Consider the area of the region between the curves \( y = x + \frac{1}{x^2} \) and \( y = x - \frac{1}{x^2} \) for \( x \geq 1 \). Is the area of this region finite or infinite? If finite, then find it.
9. Consider the solid formed by rotating \( f(x) = \cos x, -\pi/2 \leq x \leq \pi/2 \), about the \( x \)-axis.

(a) Set up an integral to compute the surface area of this solid.

(b) Using the formula \( \int \sqrt{1 + w^2}dw = \frac{1}{2} \left[w\sqrt{1 + w^2} + \ln \left(w + \sqrt{1 + w^2}\right)\right] \), compute the integral you set up in part (a).
10. Determine $a$ and $b$ so that the integral $\int \sqrt{-x^2 + 4x - 3} \, dx$ can be rewritten in the form $\int \sqrt{b^2 - (x - a)^2} \, dx$. 