22S:30/105
Statistical Methods and Computing

Wrap-Up of Normal Distributions
Intro to Regression

Lecture 5
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Standardizing values from other normal distributions

All normal distributions would be the same if we measured in units of size σ around the mean μ as center!

If x is an observation from a distribution that has mean μ and standard deviation σ, the *standardized value* of x is

$$z = \frac{x - \mu}{\sigma}$$

Standardized values are often called *z-scores*.

z-scores tell how many standard deviations the original observation is away from the mean of the distribution, and in which direction.

- If the z-score is positive, the original observation was larger than the mean μ.
- If the z-score is negative, the original observation was smaller than μ.

Example of z-scores

Recall that the distribution of systolic blood pressure of men aged 18-74 is approximately normal with $\mu = 129$ mm Hg and $\sigma = 20$ mm Hg. The standardized height is

$$z = \frac{sbp - 129}{20}$$

If a man has sbp = 157 mm Hg, his standardized sbp is

$$z = \frac{157 - 129}{20} = 1.4$$

If a man has sbp = 93 mm Hg, his standardized sbp is

$$z = \frac{93 - 129}{20} = -1.8$$

Using the standard normal distribution to compute proportions for other normal distributions

Let's use the symbol X for a variable representing the systolic blood pressure of men. What proportion of men have sbp < 100?

If a man has sbp = 100, his standardized sbp is

$$z = \frac{100 - 129}{20} = -1.45$$

According to the normal table, the proportion of values of a standard normal variable that are less than or equal to -1.45 is 0.0735.

This proportion is the same as the proportion of X values that will be less than 100.

General procedure for finding normal proportions

1. State the problem in terms of the observed variable X.
2. Standardize the value of interest x to restate the problem in terms of a standard normal variable Z. You may then wish to draw a picture to show the area under the standard normal curve.
3. Find the required area under the standard normal curve, using Table A and remembering
   - The total area under the curve is 1.0.
   - The normal distribution is symmetric.

Example

For women in the US between 18 and 74 years of age, diastolic blood pressure follows a normal distribution with mean is $\mu = 77$ mm Hg and standard deviation $\sigma = 11.6$ mm Hg.

We want to know the proportion of US women in this age group who have dbp between 60 and 100.

1. Call the variable representing a woman's dbp X, and call the specific value for an individual woman x. X has a normal distribution with $\mu = 77$ and $\sigma = 11.6$. We want to compute to compute the proportion of women such that

$$60 \leq X \leq 100$$

2. Standardize x to produce z, a draw from a standard normal distribution.

$$\begin{aligned} 60 \quad &\leq \quad X \quad \leq \quad 100 \\ \frac{60-77}{11.6} \quad &\leq \quad \tfrac{X-77}{11.6} \quad \leq \quad \frac{100-77}{11.6} \\ -1.47 \quad &\leq \quad Z \quad \leq \quad 1.98 \end{aligned}$$

3. Use Table A to find
   - the proportion of Z values ≤ -1.47, which $= .0708$
   - and the proportion of Z values ≤ 1.98, which $= .9761$.
4. So the percent of women with diastolic blood pressure between 60 and 100 is about 97.61% - 7.08% = 90.5%.

Normal calculations going the other direction

What is the value of dbp such that 10% of women have values greater than or equal to it?

1. Use Table A to find the z-score such that 10% of a standard normal population would have values greater than or equal to it.

   This is the same value such that 90% of values are less than or equal to it, namely 1.28.
2. Convert $z = 1.28$ into x.

$$\begin{aligned}\frac{x-\mu}{\sigma} &= z\\ \frac{x-77}{11.6} &= 1.28\\ x &= 77 \ + \ (11.6)(1.28)\\ x &= 91.85\end{aligned}$$

General formula for *un*standardizing a z-score:

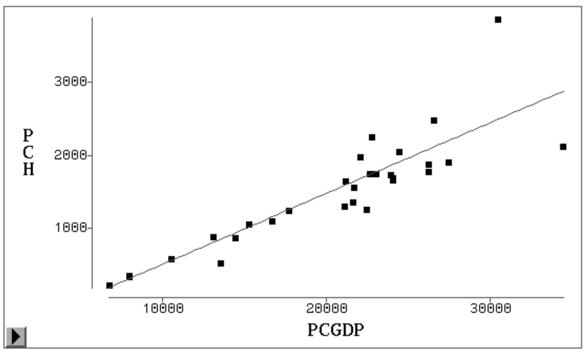
$$x \ = \ \mu + z\sigma$$

Scatterplots

- represent the relationship between two different continuous variables measured on the same subjects
- each point represents the values for one subject for the two variables

Example: data reported by the Organization for Economic Development and Cooperation on its 29 member nations in 1998

- Per capita gross domestic product (a measure of wealth of the country) is on x-axis (horizontal)
- Per capita health care expenditures is on y-axis (vertical)



We can describe the overall pattern of a scatterplot by

- form or shape
- direction
- strength

Positive and negative association

- Two variables are *positively associated* when above-average values of one tend to occur in individuals with above-average values of the other, and below-average values of both also tend to occur together.
- Two variables are *negatively associated* when above-average values of one tend to occur in individuals with below-average values of the other, and vice-versa.

Linear relationships

- The form of a relationship shown by a scatterplot is linear if the points lie in a straight-line pattern.
- The linear relationship is strong if the points lie close to a line, with little scatter.

Example: per capita health care expenditures and gross domestic product

- "individuals" studied are countries
- form of relationship is roughly linear
- direction of relationship is positive
- strength: determined by how closely the points follow a clear pattern
  - quite strong

Correlation

- a numeric measure of the direction and strength of the linear relationship between two continuous variables measured on the same subjects
- terminology and notation
  - sample correlation coefficient r

Computing the sample correlation coefficient

- We have measured two different variables X and Y on the subjects in a study.
- There are n subjects.
- Let $\bar{x}$ and $\bar{y}$ be the sample means of the two variab les.
- Denote the sample standard deviation of the x variable as s_x and the sample standard deviation of the y variable as s_y.
- Then the sample correlation coefficient is computed as

$$r = \frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{x_i - \bar{x}}{s_x} \right) \left(\frac{y_i - \bar{y}}{s_y} \right)$$

- Note that the first step in computing r is to *standardize* the measurements.
- Example: suppose X is heart rate in beats per minute and Y is body temperature in degrees Fahrenheit, and we have both heart rate and temperature measurements on $n = 10$ people.
  - The quantity

    $$\frac{x_i - \bar{x}}{s_x}$$

    is the standardized heart rate for person i
    - how many standard deviations above or below the mean herat rate person i's heart rate is
  - Standardized values are no longer in their original units (e.g., the st andardized heart rates are not in beats per minute)

- The sample correlation coefficient r is an average of the products of the standardized heart rates and temperatures for the 10 people.

Facts about correlation

- Correlation requires that both variables be quantitative, so that we can do arithmetic computations with them.
- r has no units, and, because it uses standardized values, it does not change when we change the units of measurements of x, y, or both.
  - For the same 10 people, r would not change whether we measured the heights and weights in inches and pounds or in centimeters and kilograms.
- $r > 0$ indicates a positive association between the two variable; $r < 0$ indicates a negative association
- r is always between -1 and +1
  - values of r near 0 mean a very weak linear relationship

  - values near +1 indicate a very strong positive relationship (all points lie almost exactly on a straight line)
  - values near -1 indicate a very strong negative relationship (all points lie almost exactly on a straight line)
- Correlation measures only the strength of *linear* relationships. r may be close to 0 even if the relationship between two variables is strong, if that relationship is curved.
- The sample correlation coefficient is very sensitive to outliers.
- A high correlation between two variables does not by itself imply a causal relationship.

Correlation and regression

- Correlation enables us to *assess the strength* of a linear relationship between two variables, but it does not enable us to *predict* the value of one variable for a subject for whom we know the value of the other variable.
- Prediction often is an important goal of statistical analysis.
- Example: we may wish to predict an infant's birthweight based on a laboratory measurement taken on the mother during pregnancy

Response variables and explanatory variables

- **response variable**
  - what we want to explain or predict
  - also called "dependent" or "outcome" variable
- **explanatory variable**
  - a variable that explains or influences differences in a response variable
  - also called "predictor" variables, "co-variates," or "independent" variables

- When making a scatterplot of such data:
  - response variable goes on y-axis (vertical)
  - explanatory variable goes on x-axis (horizontal)

- Note: Correlation analysis does not distinguish between response and explanatory variables.

- Example: The admissions director of the University of Iowa wants to guess how successful incoming students are likely to be.
- The high school GPA is part of each incoming student's record. The admissions director wishes to predict the student's UI GPA.
- What is the response variable and what is the explanatory variable?

Simple Linear Regression

- If a scatterplot suggests a linear relationship between 2 variables, we want to summarize the relationship by drawing a straight line on the plot.
- A *regression line* summarizes the relationship between a response variable and an explanatory variable.
  - Both variables must be quantitative.
- definition: A *regression line* is a straight line that describes how a response variable Y changes as an explanatory variable X changes.
  - often used to predict the value of Y that corresponds to a given value of X.

Recall straight lines

$$y = a + bx$$

- a : intercept; the value of Y when X = 0
- b : slope; how much Y changes when X increases by 1 unit

Least squares: choosing the "best" estimated line for a set of sample data

a and b are estimated by choosing a line as follows:

- for each observed value y_i in the sample data, compute the distance from y_i to the line
- square each of the distances
- add up all the squared distances
- choose the line that makes the sum of these squared distances the smallest

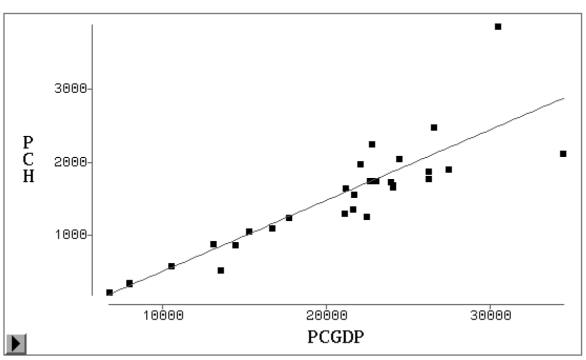
Using sample data to estimate the intercept and slope

- We will write an estimated regression line based on sample data as

$$\hat{y} = a + bx$$

- a is the estimated intercept, and b is the estimated slope
- The hat over the y means that $\hat{y}$ is the *predicted* value of the response variable, not an actual observed value

- Example: the estimated regression line for the health care expenditures and gross domestic product is



$$\hat{y} = -465.7 + 0.0968x$$

- This means that if country A has 1 unit higher PCGDP than country B, we would expect country A to have 0.0968 higher PCH than country B.

  - Since we are measuring PCH in dollars and PCGDP in dollars, this means for every additional dollar in PCGDP, we expect about a 9.7-cent increase in PCH.

- Note that it makes no sense in this problem to say that the intercept (-465.7) is the amount of per capital health care expenditure that we would expect in a country with PCGDP = 0.

- **An estimated regression line is meaningful only for the range of X values actually observed.**
  - In the PCH/PCGDP problem, this is about $8000 - 33000. The estimated intercept makes the linear relationship come out right over this range of X values.