22S:30/105
Statistical Methods and
Computing

Measures of Center, continued
Measures of Dispersion

Lecture 3
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Kate Cowles
374 SH, 335-0727
kate-cowles@uiowa.edu

The mean is meaningful only for quantitative data (either discrete or continuous).

- Example regarding a discrete variable: We hear reports such as that the average number of children per family is 1.9.

- The mean is not meaningful for nominal or ordinal data.

Exception: if a binary variable is coded as 0 and 1.

Then the arithmetic mean is the proportion of observations in the dataset that have value 1.

Example:

- An ecological study of a habitat in which 10 rare species of bird are known to have lived as of 1990

- In 1999, a naturalist is sent to spend a day in the area and to record any members of these 10 species that she observes

- A variable is coded as follows:
  - $1 =$ at least one member of the species was observed
  - $0 =$ no members of the species were observed

species	observed?
1	1
2	0
3	1
4	1
5	1
6	0
7	1
8	1
9	1
10	1

- The mean

$$\bar{x} = \frac{8}{10} = .8$$

indicates that 80% of the species were observed.

The median

The median is the 50th percentile of a set of observations.

- Values must be sorted from smallest to largest.
- If the number of observations is odd, then the median is the middle value.

$$75 \quad 80 \quad 82 \quad 88 \quad 95$$

  The median is 82.
- If the number of observations is even, then the usual way to define the median is as the **mean** of the **two** middle values.

$$75 \quad 80 \quad 82 \quad 88 \quad 95 \quad 97$$

  The median is $\frac{82+88}{2} = 85$.

The median is **not** strongly affected by a few extreme values in the dataset.

Example 1:

$$75 \quad 80 \quad 82 \quad 88 \quad 95$$

- mean = 84
- median = 82

Example 2:

$$25 \quad 80 \quad 82 \quad 88 \quad 95$$

- mean = 74
- median = 82

The median is *robust* to extreme values.

The median can be used as a measure of center for **ordinal** data as well as for discrete and continuous data.

Example: The NYC poll

```
                                  Cumulative
  city1yr   Frequency   Percent   Frequency
-------------------------------------------
  Worse        593       61.64       593
  Same         252       26.20       845
  Better       111       11.54       956
```

- 956 people answered this question regarding whether they thought the condition of the city in June, 2003, was better, worse, or the same as one year earlier.
- If the values are sorted from smallest to largest (Worse, Same, Better), then the median will be the average of the 478th and 479th values.
- We can use the cumulative frequencies in the table to figure out what these have to be. They are both in category "Worse."
- Thus the median is Worse.

The mode

- The mode of a set of values is the value that occurs most frequently.
- Example: in the NYC poll data, the mode of the "city1yr" variable is Worse.
- Example: There is no mode in the birthweights data, because no value occurs more than once.
- There may be more than one mode in a set of values.
- The mode may be reported for **all** types of data.

When is each measure of central tendency appropriate?

Depending on data type

- Nominal data
  - mode only
  - possible exception: binary data coded 0 and 1
- Ordinal data
  - mode or median
- Quantitative data
  - mean, median, or mode

Depending on the shape of the distribution
of values (quantitative variables)

- if the shape is approximately symmetric and has only one mode
  - mean and median will be close in value
  - mean is typically reported

Example: the body temperature data
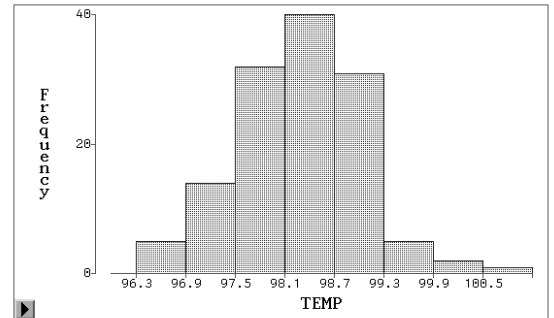
From a statistical computer package:
  - mean = 98.24
  - median = 98.3

- if the distribution is highly skewed
  - if skewed to the right, mean will be larger than median
  - if skewed to the left, mean will be smaller than median
  - mean may not be a "typical" value
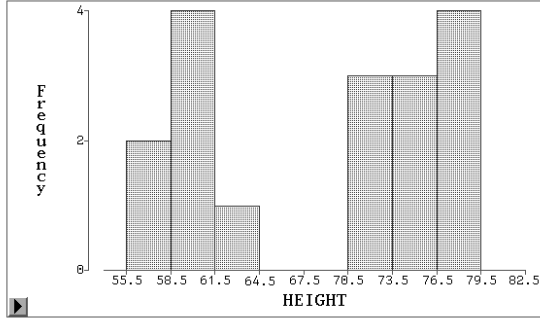
Example: the billionaire data

From a statistical computer package:
  - mean = 2.7 billion
  - median = 1.8 billion

- if the distribution has more than one mode
  - neither the mean nor the median may be representative values
  - may be best to report all modes and/or to display a graph
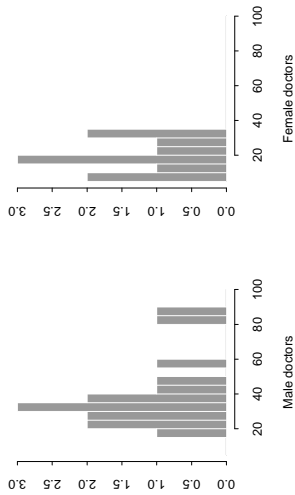 - may occur if two or more different subgroups are represented in the sample

Example:

From a statistical computer package:

– mean = 69.0
– median = 72.0

In getting the "overall picture" of quantitative data, the spread is just as important as the center of the data.

Numerical measures of dispersion

- the range
- the interquartile range
- the standard deviation

The range

- The range is the difference between the largest and the smallest observations.

- For the male Swiss doctors,
  - largest value = 86
  - smallest value = 20
  - range = $86 - 20 = 66$

- For the female Swiss doctors,
  - largest value = 33
  - smallest value = 5
  - range = $33 - 5 = 28$

The range shows the full spread of the data, but may be exaggerated if the largest and/or smallest values are atypical (outliers)

- Example: the 1992 billionaire data
  - With Bill Gates:
    range = 37 - 1 = 36 billion
  - If Bill were deleted:
    range = 24 - 1 = 23 billion

- Example: the male Swiss doctors data
  - With the largest two values
    range = 86 - 20 = 66 billion
  - If the two largest values were deleted:
    range = 59 - 20 = 39 billion

- So additional measures are needed to give a more complete picture of the spread of values.

The quartiles and the interquartile range

- The *first quartile* is the same as the 25th percentile
  - one quarter of the observations in a dataset have values less than or equal to the 1st quartile, and the other three quarters have values greater than or equal to the first quartile

- The *third quartile* is the same as the 75th percentile
  - three quarters of the observations in a dataset have values less than or equal to the 3rd quartile, and the other one quarter have values greater than or equal to the 3rd quartile

- The interquartile range (IQR) is the difference between the 3rd and 1st quartiles

- For the male Swiss doctors,
  - third quartile = 50
  - first quartile = 27
  - IQR = 50 - 27 = 23

- For the female Swiss doctors,
  - third quartile = 29
  - first quartile = 14
  - IQR = $29 - 14 = 15$

- For the 1992 billionaires,
  - third quartile = 3 billion
  - first quartile = 1.3 billion
  - IQR = $3 - 1.3 = 1.7$ billion

The IQR is considered less sensitive to outliers than the range.

- Example: the 1992 billionaire data
  - With Bill Gates:
    IQR $= 3 - 1.3 = 1.7$ billion
  - If Bill were deleted:
    IQR $= 2.9 - 1.3 = 1.6$ billion

- However, in a small dataset, deletion of a few outliers may affect the IQR substantially.

- Example: the male Swiss doctors
  - IQR with the two largest values included:
  - IQR $= 50 - 27 = 23$
  - IQR with the two largest values deleted:
  - IQR $= 37 - 27 = 10$

The five-number summary

- The five-number summary provides a reasonably-complete numeric summary of the center and dispersion of a set of values.

- The five-number summary consists of
  - the minimum value
  - the first quartile
  - the median
  - the third quartile
  - the maximum value

The five-number summary for the billionaire data may be extracted from the following computer output:

```
              Quantiles(Def=5)

    100% Max      37        99%      14
     75% Q3        3        95%     6.2
     50% Med     1.8        90%     4.5
     25% Q1      1.3        10%     1.1
      0% Min       1         5%       1
                            1%       1

    Range        36
    Q3-Q1       1.7
    Mode          1
```
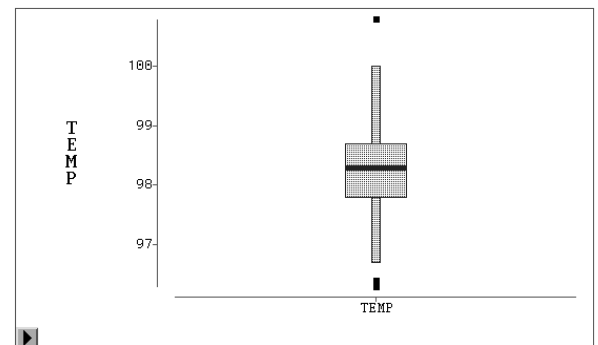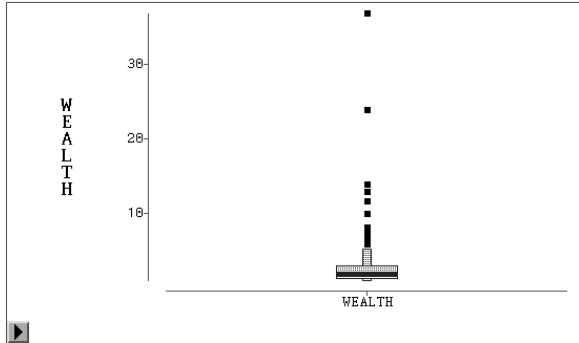
Boxplots

- are used to summarize the distribution of a continuous variable

- box extends from 1st quartile to 3rd quartile of data

- line in middle of box marks 50th percentile

- "whiskers" sticking out of box extend to *adjacent values*
  - adjacent values are most extreme observations that are not farther away from the edge of the box than 1.5 times the height of the box
- points farther out than the adjacent values are considered *outliers*
  - represented by circles or squares
  - probably are not typical of the rest of the data

The standard deviation

- The standard deviation measures spread by looking at how far the observations are from their mean.

- Example: quiz scores

  75   80   82   88   95

  The mean is
  $$\bar{x} = \frac{75 + 80 + 82 + 88 + 95}{5}$$
  $$= 84$$

  points.

- We want a measure of typical distance between an individual value and this mean.

An idea that won't work for measuring the spread: take the average of the "deviations" of the individual observations from the mean.

Observed Value	Deviation from mean		Squared deviation	
75	75 - 84 =	-9	$(-9)^2 =$	81
80	80 - 84 =	-4	$(-4)^2 =$	16
82	82 - 84 =	-2	$(-2)^2 =$	4
88	88 - 84 =	4	$4^2 =$	16
95	95 - 84 =	11	$11^2 =$	121
sum		0		238

Because the sum of the deviations is always 0, the average deviation is always 0!

Solution: Square the individual deviations before adding them up!

The variance and the standard deviation

- The variance s^2 is the sum of the squared deviations divided by one less than the number of observations.

$$s^2 = \frac{\Sigma_{i=1}^{n}(x_i - \bar{x})^2}{n - 1}$$
$$= \frac{238}{4} = 59.5$$

  - We can think of the variance roughly as the average of the squared deviations.

- The standard deviation is the square root of the variance.
  $s = \sqrt{59.5} = 7.71$ points.

Facts about the standard deviation
s

- s measures the spread of values around the *mean*
  - thus s should be used as a measure of dispersion only when mean has been chosen as the measure of center

- s is always greater than 0 unless all the observations have the same value

- s has same units of measurement as original observations

- s is sensitive to extreme observations
  - like the mean

- s is the most commonly-used measure of dispersion (is often used when it is not the best choice!)

The mean and standard deviation together provide a reasonable numeric summary of a set of values if the distribution is approximately **symmetric**.

- Example: the body temperature data

```
    Variable    N          Mean        Std Dev
    -------------------------------------------
    TEMP       130     98.2492308     0.7331832
```

- Example of inappropriate use of $\bar{x}$ and s to summarize a distribution: the billionaire data

```
    Analysis Variable : WLTH

     N          Mean        Std Dev
    -------------------------------
    233      2.6815451      3.3188403
    -------------------------------
```