

22S:105 Statistical Methods and Computing

More on t-tests

Lecture 16
Mar. 27, 2013

```
data normtemp ;
infile 'normtemp.dat' ;
input temp gender heart ;
run ;

proc means n mean stddev clm alpha = .05 ;
var temp ;
run ;
```

Variable	N	Mean	Std Dev	Lower 95.0% CLM	Upper 95.0% CLM
temp	130	98.249	0.733	98.122	98.376

Note that:

- The 95% confidence interval for μ does not contain 98.6.
- The p-value is less than .05, so we can reject the null hypothesis.

SAS for one-sample t-tests

- SAS automatically does a two-sided test

$$H_0 : \mu = \mu_0$$

$$H_a : \mu \neq \mu_0$$

Example: Using the “normtemp.dat” data on body temperatures measured on 130 healthy adults, we will test the hypotheses

$$H_0 : \mu = 98.6$$

$$H_a : \mu \neq 98.6$$

at the .05 significance level.

Example 2: We will use the same dataset to test a hypothesis regarding heart rates, namely:

$$H_0 : \mu = 73$$

$$H_a : \mu \neq 73$$

at the .05 significance level.

Analysis Variable : HEART

N	Mean	Std Dev	Lower 95.0% CLM	Upper 95.0% CLM
130	73.7615385	7.0620767	72.5360699	74.9870071

Note that:

- The 95% confidence interval for μ does contain 73.
- The p-value is greater than .05, so we cannot reject the null hypothesis.

One-sample t-tests using proc univariate

```
data normtemp ;
infile '/group/ftp/pub/kcowles/datasets/normtemp.dat' ;
input temp gender heart ;
format gender sexfmt. ;
run ;

proc univariate mu0 = 98.6 data = normtemp ;
var temp ;
run ;
```

The UNIVARIATE Procedure
Variable: temp

Tests for Location: Mu0=98.6

Test	-Statistic-	-----p Value-----
Student's t	t -5.45482	Pr > t <.0001
Sign	M -21	Pr >= M 0.0002
Signed Rank	S -1963	Pr >= S <.0001

Paired samples

- We are interested in the unknown population means μ_1 and μ_2 of two different populations.
- In our sample, each observation drawn from the first population is matched up with an observation drawn from the second population.

Two-sample t-tests

So far we have talked about drawing inference about a single population mean μ based on data contained in one sample drawn from that population.

Now we will consider procedures for comparing two *different* population means.

There are different procedures depending on whether the samples are

- paired
- independent

- *self-pairing*: two measurements are taken on each subject

Example:

- systolic blood pressure (sbp) upon entry into a clinical study
- sbp after 1 month on treatment

The population means of interest are

- μ_1 = mean sbp of untreated patients of this type
- μ_2 = mean sbp of patients of this type after 1 month of treatment with the study regimen
- The question of interest is whether the treatment lowers blood pressure, i.e. is $\mu_2 < \mu_1$?

- *matched pairs*: investigator matches each subject in one treatment group with one subject in another treatment group so that members of a pair are as alike as possible

The population means of interest are

- μ_1 = mean response (say sbp at 1 month) of patients receiving treatment 1
- μ_2 = mean response of patients receiving treatment 2
- The question of interest is whether $\mu_1 = \mu_2$

come OC users. This subgroup will be the study sample.

- Measure the sbp of the study sample at the follow-up visit.
- We will compare the baseline and follow-up sbps of the women in the study sample.

Paired t-test

To carry out the hypothesis test of interest, we apply one-sample procedures to the *differences* between values measured on members of each pair.

Example:

- We are interested in whether the use of oral contraceptive (OC) drugs affects the level of systolic blood pressure (sbp) in women.
- We identify a group of nonpregnant, premenopausal women aged 16-49 from a prepaid health plan who are not currently OC users and measure their sbp, which we will refer to as baseline sbp.
- We rescreen these women 1 year later to ascertain a subgroup who have remained non-pregnant throughout the year and have be-

We will do a two-sided test, because we do not know in advance whether to expect μ_1 (mean sbp in OC users) to be higher or lower than μ_2 (mean sbp in non-users).

$$H_0 : \mu_1 = \mu_2$$

$$H_a : \mu_1 \neq \mu_2$$

or equivalently:

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_a : \mu_1 - \mu_2 \neq 0$$

or equivalently:

$$H_0 : \delta = 0$$

$$H_a : \delta \neq 0$$

where δ denotes $\mu_1 - \mu_2$.

We will use the *observed differences* between the before and after values observed on each woman as our data to carry out the hypothesis test regarding δ at the .05 significance level.

```
data sbpoc ;
infile '/group/ftp/pub/kcowles/datasets/sbpoc.dat' ;
input sbpnooc sbpoc ;
diff = sbpoc - sbpnooc ;
run ;

proc print ;
run ;
```

OBS	SBPNOOC	SBPOC	DIFF
1	115	128	13
2	112	115	3
3	107	106	-1
4	119	128	9
5	115	122	7
6	138	145	7
7	126	132	6
8	105	109	4
9	104	102	-2
10	115	117	2

We will compute the sample mean of the d_i s

$$\bar{d} = \frac{\sum_i^n d_i}{n}$$

and the sample standard deviation of the d_i s

$$s_d = \sqrt{\frac{\sum_i^n (d_i - \bar{d})^2}{n - 1}}$$

```
proc means data = sbpoc ;
var diff ;
run ;
```

Analysis Variable : DIFF

N	Mean	Std Dev	Minimum	Maximum
10	4.800	4.5655716	-2.0000	13.0000

Then the t statistic is

$$t = \frac{\bar{d} - \delta_0}{s_d / \sqrt{n}}$$

From our data,

$$\begin{aligned} \bar{d} &= 4.80 \\ s_d &= 4.566 \\ t &= \frac{4.80}{4.566 / \sqrt{10}} \\ &= 3.32 \end{aligned}$$

Using Table A, we see that the value that cuts off the upper .025 area under a t distribution with 9 degrees of freedom is 2.262.

Because $3.32 > 2.262$ (our result is more extreme than the required cutoff), we can reject the null hypothesis at the .05 level.

We could use SAS to find the exact p-value, which is 0.0089.

Note that the one-sample t-test in `proc univariate` by default tests the null hypothesis that $\mu = 0$.

```
proc univariate data = sbpoc ;
var diff ;
run ;
```

```
      The UNIVARIATE Procedure
      Variable:  temp
```

```
Tests for Location: Mu0=0
```

Test	-Statistic-	-----p Value-----
Student's t	t 3.324651	Pr > t 0.0089
Sign	M 3	Pr >= M 0.1094
Signed Rank	S 24	Pr >= S 0.0117