22S:30/105 Statistical Methods and Computing

Linear Regression, continued

Lecture 6
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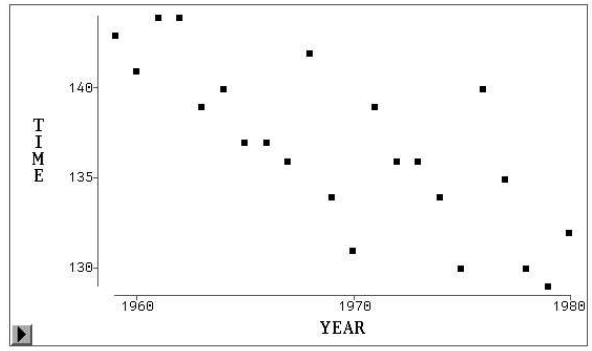
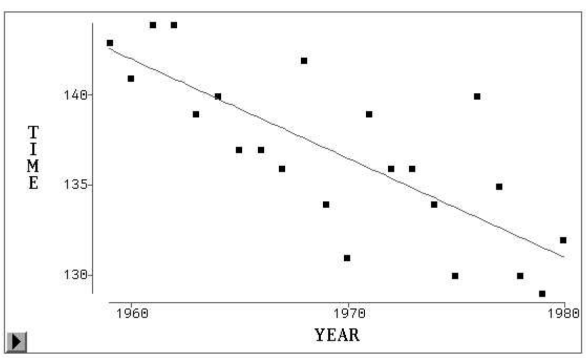
Another example: Men's winning times in the Boston Marathon, 1959-80





$$\hat{y} = 1221.05 - 0.5505x$$

What does this equation tell us?

Prediction using an estimated regression line

Example: What is the predicted PCH for a country with PCGDP = \$20,000?

$$\begin{aligned}\hat{y} &= -465.7 + 0.0968(20000) \\ &= 2401.70\end{aligned}$$

What is the predicted winning time for the Boston marathon in 1965?

How well does the regression line predict the response variable?

- The **coefficient of determination** or R^2
  - When there is only 1 explanatory variable, $R^2 = r^2$ — the square of the correlation coefficient r between the response variable and the explanatory variable
  - the proportion of the variability among the observed values of the response variable that is explained by the linear regression
- Example: in the OECD health care expenditures data, $R^2 = 0.764$
  - 76.4% of the variability in per capita health care expenditures is explained by PCGDP

Notation

Recall:

- y_i is the observed value of the response variable for subject i
- $\hat{y}_i$ is the value predicted by the regression line for subject i

$$\hat{y}_i = a + bx_i$$

Residuals

- A **residual** is the difference between an observed value and a predicted value of the response variable.

$$r_i = y_i - \hat{y}_i$$

- There is a residual for each data point.
- The residual for the ith observation will be positive if the observed value lies above the estimated regression line.
- The mean of the residuals from a least-squares fit is always 0

Example: the OECD health care expenditures data

$$\hat{y} = -465.7 + 0.0968x$$

- The predicted score for the US, for which $x_1 = 30,514$ dollars is

$$\hat{y}_1 = -465.7 + 0.0968(30514) = 2488$$

- The actual value of PCH for the US is \$3898.
- The residual for the US is positive because the data point lies above the regression line.

$$r_i = 3898 - 2488 = 1410$$

Residual plots

- A residual plot is a scatterplot of the regression residuals against the predicted values of the response variable.
- Residual plots help
  - assess fit of a regression line
  - look for violations of the assumptions of linear regression and for problematic data points

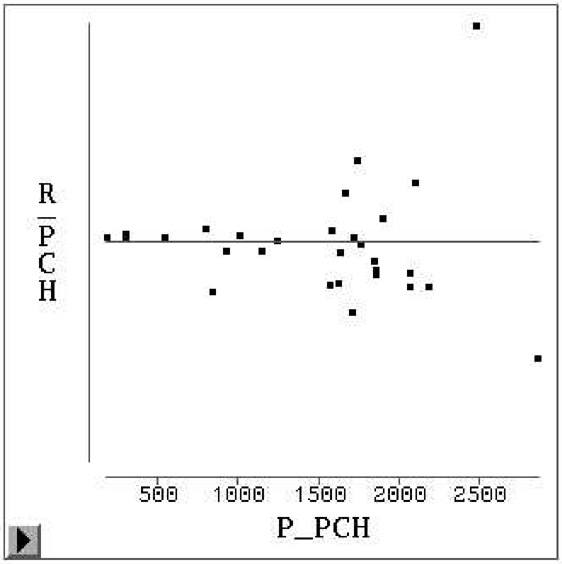
Things to watch for in a residual plot

- a random scatter of points
  - This is what you *want* to see.
- A curved pattern
  - indicates that the relationship between the response variable and the explanatory variable is *not* linear
  - violation of an assumption
- increasing or decreasing spread around the zero line
  - indicates violation of the assumption that σ is the same in all the subpopulations

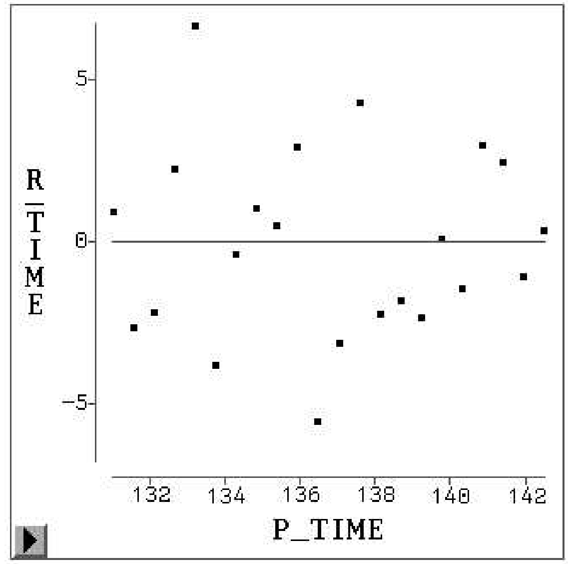
- individual points with large residuals
  - outliers in the vertical direction
  - these points are not well described by the regression equation
- individual points that are extreme in the horizontal direction (unusual values of explanatory variable)
  - These may be influential observations.

Idealized patterns in residual plots

The residual plot for the OECD health care expenditures data



The residual plot for the Boston marathon data



Outliers and influential observations

- Outlier: an observation that lies outside the overall pattern of the other observations.
- Influential observation: an observaion is *influential* for a statistical calculation if removing it would markedly change the result of the calculation. Points that are outliers in the x direction (have unusual values of the explanatory variable) are often influential in computing the least-squares regression line.

- In the OECD data, the US and Luxemburg are both outliers.
- The US is influential.
  - With the US included in the analysis, $R^2 = 0.764$. If the US is deleted, R^2 increases to 0.846.

Facts about least-squares regression

- Keep straight which is the response variable and which is the explanatory variable. If they are switched, a different regression line results.
- The correlation coefficient r and the slope b of the regression line are closely related.
  - They always have the same sign (both positive, or both negative, or both zero).
  - The slope of the regression line is

    $$b = r\frac{s_y}{s_x}$$

    This means that a change of one standard deviation in x corresponds to a change of r standard deviations in y.
- How large or small the slope is does *not* indicate how strong the relationship between the response variable and the explanatory variable is.

  - The magnitude of the slope depends on the units in which we measure both variables.
    * Example: If we measured the winning times in the Boston marathon in hours instead of minutes, the slope would be -0.0092 instead of -0.55, but the relationship between winning time and year of race would be the same!
  - The correlation coefficient r is needed to quantify the strength of the relationship.
- But the correlation coefficient is not enough to enable us to *predict* the value of a response variable if we know the value of an explanatory variabl.e
  - For prediction, we need the regression equation.

- The least-squares regression line always passes through the point $(\bar{x}, \bar{y})$.
- The square of the correlation coefficient, r^2, is the fraction of the variation in the values of y that is explained by the least-squares regression line.
  - How much better are we able to predict y because we know x?

Caveats about regression and correlation

- It usually doesn't make sense to try to use the regression equation to predict for values of the explanatory variable outside the range of observed data.
- Correlations based on averaged data are usually too large to be applicable to individuals.
  - Example: Correlation between national female literacy rates and national infant mortality rates in countries in Latin America
- Lurking variables
  - one or more variables that have an important effect on the relationship among variables under study but that are not considered in the study