22S:105
Statistical Methods and Computing

Two kinds of two-sample t-tests

Lecture 16
Mar. 11, 2019

Two-sample t-tests

So far we have talked about drawing inference about a single population mean μ based on data contained in one sample drawn from that population.

Now we will consider procedures for comparing two *different* population means.

There are different procedures depending on whether the samples are

- paired
- independent

Paired samples

- We are interested in the unknown population means μ_1 and μ_2 of two different populations.
- In our sample, each observation drawn from the first population is matched up with an observation drawn from the second population.

- *self-pairing*: two measurements are taken on each subject

  Example:

  – systolic blood pressure (sbp) upon entry into a clinical study
  – sbp after 1 month on treatment

  The population means of interest are

  – μ_1 = mean sbp of untreated patients of this type
  – μ_2 = mean sbp of patients of this type after 1 month of treatment with the study regimen
  – The question of interest is whether the treatment lowers blood pressure, i.e. is $\mu_2 < \mu_1$?

- *matched pairs*: investigator matches each subject in one treatment group with one subject in another treatment group so that members of a pair are as alike as possible

  The population means of interest are

  - μ_1 = mean response (say sbp at 1 month) of patients receiving treatment 1
  - μ_2 = mean response of patients receiving treatment 2
  - The question of interest is whether $\mu_1 = \mu_2$

Paired t-test

To carry out the hypothesis test of interest, we apply one-sample procedures to the *differences* between values measured on members of each pair.

Example:

- We are interested in whether the use of oral contraceptive (OC) drugs affects the level of systolic blood pressure (sbp) in women.
- We identify a group of nonpregnant, premenopausal women aged 16-49 from a prepaid health plan who are not currently OC users and measure their sbp, which we will refer to as baseline sbp.
- We rescreen these women 1 year later to ascertain a subgroup who have remained nonpregnant throughout the year and have be-

come OC users. This subgroup will be the study sample.

- Measure the sbp of the study sample at the follow-up visit.
- We will compare the baseline and follow-up sbps of the women in the study sample.

We will do a two-sided test, because we do not know in advance whether to expect μ_1 (mean sbp in OC users) to be higher or lower than μ_2 (mean sbp in non-users).

$$H_0 : \mu_1 = \mu_2$$
$$H_a : \mu_1 \neq \mu_2$$

or equivalently:

$$H_0 : \mu_1 - \mu_2 = 0$$
$$H_a : \mu_1 - \mu_2 \neq 0$$

or equivalently:

$$H_0 : \delta = 0$$
$$H_a : \delta \neq 0$$

where δ denotes $\mu_1 - \mu_2$.

We will use the *observed differences* between the before and after values observed on each woman as our data to to carry out the hypothesis test regarding δ at the .05 significance level.


```
data sbpoc ;
infile '/group/ftp/pub/kcowles/datasets/sbpoc.dat' ;
input sbpnooc  sbpoc ;
diff = sbpoc - sbpnooc ;
run ;

proc print ;
run ;

OBS     SBPNOOC     SBPOC     DIFF
 1        115        128       13
 2        112        115        3
 3        107        106       -1
 4        119        128        9
 5        115        122        7
 6        138        145        7
 7        126        132        6
 8        105        109        4
 9        104        102       -2
10        115        117        2
```


We will compute the sample mean of the d_is

$$\bar{d} = \frac{\Sigma_i^n d_i}{n}$$

and the sample standard deviation of the d_is

$$s_d = \sqrt{\frac{\Sigma_i^n (d_i - \bar{d})^2}{n-1}}$$

```
proc means data = sbpoc ;
var diff ;
run ;

Analysis Variable : DIFF

N      Mean       Std Dev      Minimum      Maximum
-----------------------------------------------
10     4.800      4.5655716    -2.0000      13.0000
-----------------------------------------------
```


Then the t statistic is

$$t = \frac{\bar{d} - \delta_0}{s_d/\sqrt{n}}$$

From our data,

$$\begin{aligned} \bar{d} &= 4.80 \\ s_d &= 4.566 \\ t &= \frac{4.80}{4.566/\sqrt{10}} \\ &= 3.32 \end{aligned}$$

Using Table c, we see that the value that cuts off the upper .025 area under a t distribution with 9 degrees of freedom is 2.262.

Because $3.32 > 2.262$ (our result is more extreme than the required cutoff), we can reject the null hypothesis at the .05 level.

We could use SAS to find the exact p-value, which is 0.0089.

Note that the one-sample t-test in `proc univariate` by default tests the null hypothesis that $\mu = 0$.

```
proc univariate data = sbpoc ;
var diff ;
run ;
```

```
                         The UNIVARIATE Procedure
                              Variable:  temp

                        Tests for Location: Mu0=0

             Test           -Statistic-    -----p Value------

             Student's t    t  3.324651    Pr > |t|    0.0089
             Sign           M         3    Pr >= |M|   0.1094
             Signed Rank    S        24    Pr >= |S|   0.0117
```