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STAT:2010/4200, Statistical Methods and Computing
Spring 2016, Instructor: Cowles
Midterm 2

Name: Solutions Course no. (2010 or 4200) _____

Show your work on any problems that involve calculations. If your answer to a multiple choice or true-false question would vary under different conditions, write an explanation. I will grade on a curve and will give partial credit wherever possible.

The giant fruit bat, also known as the Indian flying fox, is the largest species of bats. It has a very large wingspan, which allows it to fly and to wrap itself to keep warm.

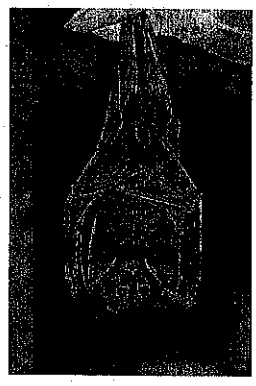

Figure 1: Giant fruit bat

Figure 1 was downloaded from
http://cincinnatizoo.org/blog/animals/indian-flying-fox-giant-fruit-bat/

1. Suppose that

   - the distribution of wingspans in the population of giant fruit bats is normal
   - the population mean μ is unknown
   - the standard deviation σ is known to be 4 inches

   Researchers plan to measure the wingspans in a sample of $n = 16$ giant fruit bats and to use the data to test the following hypotheses:

   $$H_0: \ \mu \leq 64$$
   $$H_A: \ \mu > 64$$

   (a) What is the sampling distribution of $\bar{X}$ if H_0 is true? Give the name of the distribution and the numeric values of its mean and standard deviation.

3 Normal, mean = 64

$\sigma_{\bar{x}} = \frac{4}{\sqrt{4}} = 1$

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(b) The researchers wish to carry out their hypothesis test at significance level $\alpha = 0.05$. The significance level is (circle one):

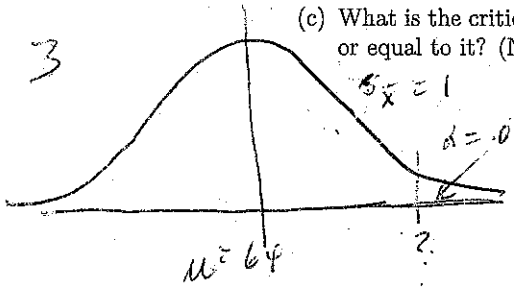
i. The probability that the result is important.

ii. The level of risk that the researchers are willing to take of incorrectly rejecting H_0.

iii. The level of risk that the researchers are willing to take of incorrectly rejecting H_A.

iv. The probability, computed assuming that H_0 is true, that the test statistic would take a value as extreme as, or more extreme than, the value actually observed.

(c) What is the critical value of $\bar{x}$ such that H_0 should be rejected for all values greater than or equal to it? (Numeric answer; show your work.)



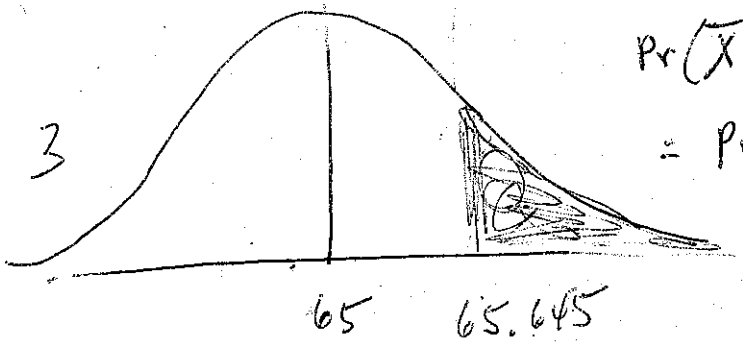
z^* to cut off upper .05 is 1.645

$$\bar{x} = \mu + z^* \sigma_{\bar{x}}$$
$$= 64 + 1.645(1)$$
$$= 65.645$$

(d) What is the sampling distribution of $\bar{X}$ if the true value of μ in the population is 65 inches? Give the name of the distribution and the numeric values of its mean and standard deviation.

Normal mean = 65
$\sigma_{\bar{x}} = 1$

(e) What is the power of the researchers' test against the alternative hypothesis $H_A : \mu = 65$? (Numeric answer; show your work.)



$$\Pr(\bar{X} \geq 65.645 \mid \mu = 65)$$
$$= \Pr\left(Z \geq \frac{65.645 - 65}{1}\right)$$
$$= 1 - \Pr(Z \leq 0.645)$$
$$= 1 - .74$$
$$= .26$$

2. Suppose that the researchers in the previous problem were interested in a two-sided hypothesis test rather than a one-sided test, that is, they wanted to test:

$$H_0 : \mu = 64$$
$$H_A : \mu \neq 64$$

They still are assuming that individual values in the population follow a normal distribution with standard deviation $\sigma = 4$ inches, and they still have $n = 16$ observations.

(a) Suppose they obtain $\bar{x} = 66.2$ inches. Compute a 95% confidence interval for the population mean μ. (Numeric answer; show your work.)

3 $\bar{x} \pm 1.96(1) = (64.24, 68.16)$

66.20
1.96
64.24

(b) Based on your confidence interval, should the researchers reject H_0? Briefly explain why or why not.

2 Yes, because the value from H_0 (64) is not in the 95% c.i., they should reject H_0 at significance level $\alpha = .05$.

(c) The correct interpretation of the 95% confidence interval is (circle one):

i. There is 95% probability that μ is in the interval.

2 ii. There is 95% probability that $\bar{x}$ is in the interval.

iii. We have used a procedure which, in 95% of random samples, produces an interval that contains the true μ.

iv. We have used a procedure which, in 95% of random samples, produces an interval that contains the true $\bar{x}$.

v. We have used a procedure that produces an interval that contains 95% of individual values from the population.

3. One-sample t confidence intervals and t-tests are appropriate when (circle **all** of the following conditions that apply):

2 1/2 (a) The population distribution is assumed to be normal, or approximately so.

(b) The population mean μ is known.

(c) The population standard deviation σ is known.

(d) There are outliers in the sample.

(e) The variable of interest is binary.

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4. Choose a young adult (aged 25 to 29) at random. The probabilities are

   - 0.13 that the person chosen did not complete high school
   - 0.31 that the person has a high school diploma but no further education
   - 0.29 that the person has at least a college degree

   (a) What must be the probability that a randomly chosen young adult has some education beyond high school but does not have a college degree? (Numeric answer; show your work.) This is the only remaining category, so

   $1 - (0.13 + 0.31 + 0.29) = 0.27$

   (b) What is the probability that a randomly chosen young adult has at least a high school education? (Numeric answer; show your work.)

   $1 - 0.13 = 0.87$

5. We have measurements of the diameters in micrometers of 41 lymphocyte cells. Here is a summary from proc means of these measurements:

Analysis Variable : diam

type	N Obs	N	Mean	Std Dev	Std Error
lymph	41	41	6.95	1.60	0.25

Analysis Variable : diam

type	N Obs	Lower 80% CL for Mean	Upper 80% CL for Mean
lymph	41	6.62	7.28

   (a) What kind of confidence interval does proc means calculate? (Circle one.)

   i. z interval
   ii. t interval
   iii. neither of the above

   (b) The above output shows an 80% confidence interval. Would a 90% interval be wider or narrower than the 80% one?

   90% interval would be wider

   (c) Using values from the SAS output and possibly from a table from your book, show how the 80% confidence interval was calculated.

   $\bar{x} \pm t^* s.e.$

   $6.95 \pm 1.303(0.25)$

   from t with $(41-1) = 40$ degrees of freedom