

```

> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected

> u1:=[1,2,3];
                                u1 := [1, 2, 3]

> u2:=[-2,1,0];
                                u2 := [-2, 1, 0]

> dotprod(u1,u2);
                                0

> v:=[0,4,2];
                                v := [0, 4, 2]

> normsq:=proc(w) w[1]^2+w[2]^2+w[3]^2 end;
                                normsq := proc(w) w[1]^2 + w[2]^2 + w[3]^2 end proc;

> normsq(u1);
                                14

> normsq(u2);
                                5

> dotpr:=proc(w,ww) w[1]*ww[1]+w[2]*ww[2]+w[3]*ww[3] end;
                                dotpr := proc(w, ww) w[1]*ww[1] + w[2]*ww[2] + w[3]*ww[3] end proc;

> dotpr(v,u1);
                                14

> dotpr(v,u2);
                                4

> ProjOfVOnU1:=dotpr(v,u1)/normsq(u1)*u1 ;
                                ProjOfVOnU1 := [1, 2, 3]

> ProjOfVOnU2:= dotpr(v,u2)/normsq(u2)*u2;
                                ProjOfVOnU2 :=  $\left[ \frac{-8}{5}, \frac{4}{5}, 0 \right]$ 

> ProjOfVOnU:=ProjOfVOnU1 + ProjOfVOnU2;
                                ProjOfVOnU :=  $\left[ \frac{-3}{5}, \frac{14}{5}, 3 \right]$ 

```

As a test of whether the orthogonal projection above really "pulls everything U-like from v", let's check that the remaining part of v is orthogonal to U; that is, check that $v - (\text{ProjOfVOnU})$ is orthogonal to both u1 and u2.

```
> N:=v-ProjOfVOnU;
```

$$N := \begin{bmatrix} \frac{3}{5} & \frac{6}{5} & -1 \end{bmatrix}$$

```
> dotpr(N,u1);
```

0

```
> dotpr(N,u2);
```

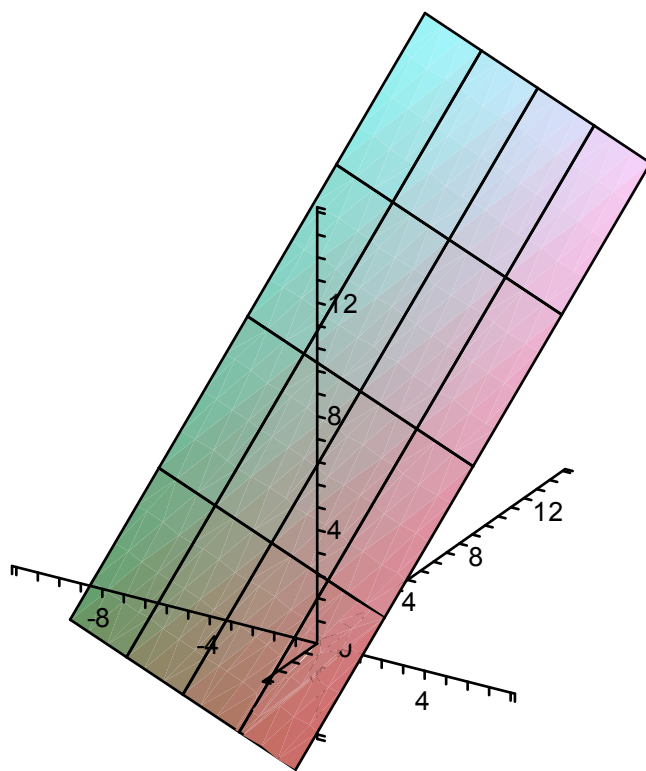
0

Now let's see what the geometry looks like.

```
> with(plots):
```

```
> SubspaceU:=plot3d(expand(s*u1+t*u2), s=-1..5, t=-1..5, grid=[5,5]):
```

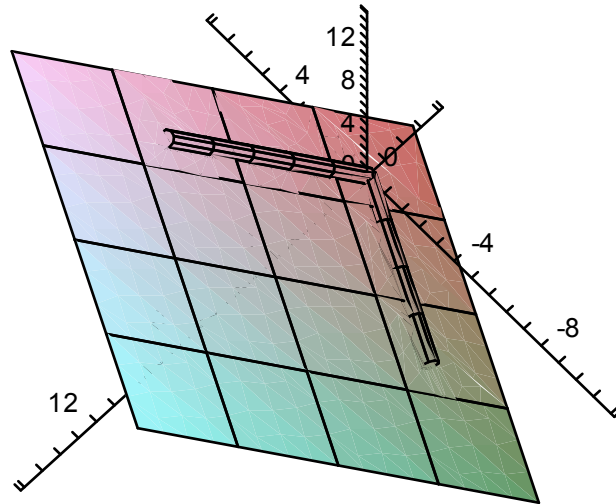
```
> display(SubspaceU);
```



```
> VectorU1:=tubeplot(expand(t*u1), t=0..3, radius=.3, numpoints=6):
```

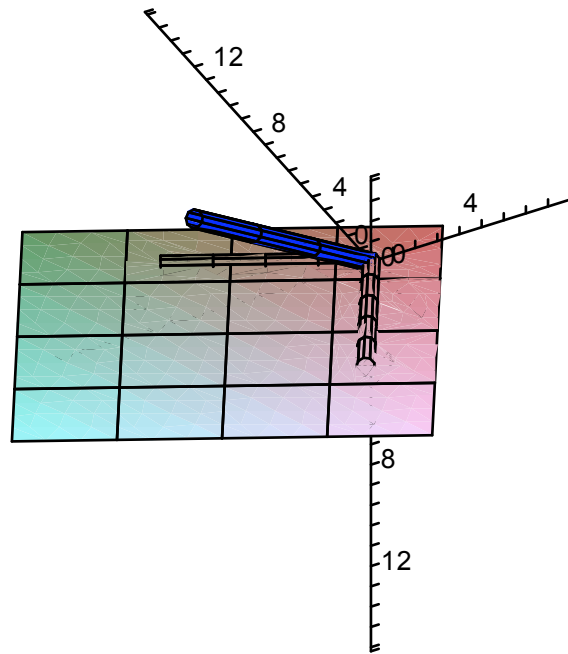
```
> VectorU2:=tubeplot(expand(t*u2), t=0..3, radius=.3, numpoints=5):
```

```
> display({SubspaceU, VectorU1, VectorU2}, axes=normal);
```

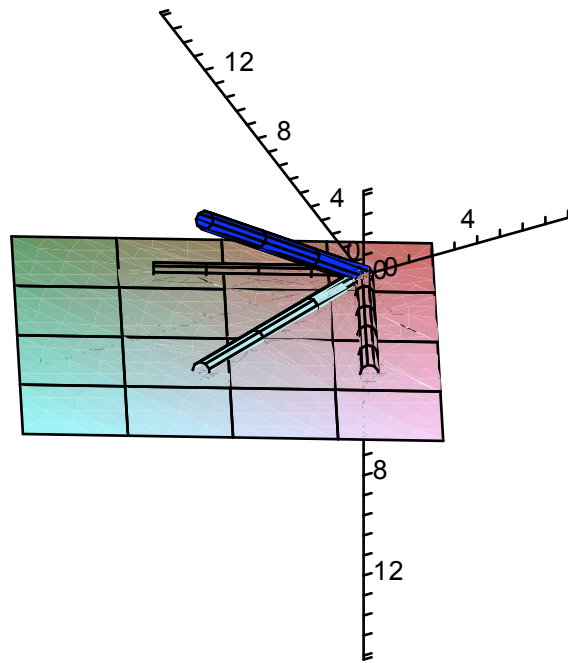


```
> VectorV:=tubeplot(expand(t*v), t=0..3, radius=.3, numpoints=4,  
color=blue):
```

```
> display({SubspaceU, VectorU1, VectorU2, VectorV}, axes=normal);
```



```
> VectorProjV:=tubepLOT(expand(t*ProjOfVOnU), t=0..3, radius=.3,
numpoints=4, color=turquoise);
> display({SubspaceU, VectorU1, VectorU2, VectorV, VectorProjV},
axes=normal);
```



```

> VectorN:=tubepoint(expand(t*N+3*ProjOfVOnU), t=0..3, radius=.3,
numpoints=4, color=pink);
> display({SubspaceU, VectorU1, VectorU2, VectorV, VectorProjV, VectorN},
axes=normal);

```

