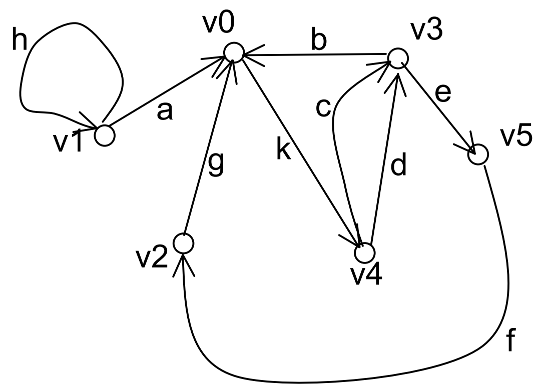
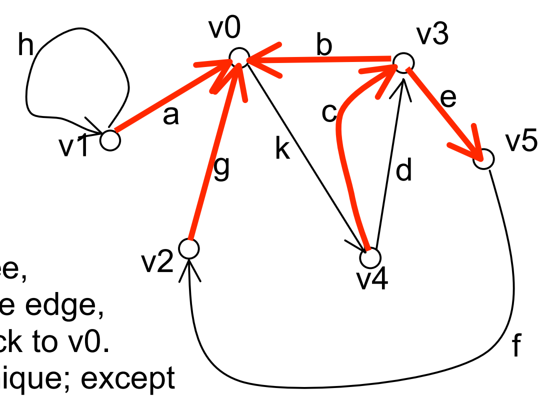
Example of calculating the fundamental group of a graph G



Note: This graph includes a loop and some vertices connected by more than one edge; the method of calculating $\pi_1(G)$ for graphs is not bothered by these.

Step 0: List the vertices: v0, v1, v2, v3, v4, v5

Step 1: List the edges, and give each an orientation: a, b, c, d, e, f, g, h, k.
Note each "edge" is a path in G. Denote the *reverse* of each edge as aˆ, bˆ, cˆ, dˆ, eˆ, fˆ, gˆ, hˆ , kˆ .



Step 2: Identify a maximal tree, **T**, in G.

Step 3: Pick one vertex to be the basepoint.
(We will use v0.)

Step 4: For each edge NOT in the maximal tree, construct a path from v0 to the beginning of the edge, and another path from the end of the edge back to v0.
(Note: Because **T** is a tree, these paths are unique; except for duplicating a path (forward, back, forward again) there is only one way to travel in **T** from v0 to another vertex.) This gives a set of loops in G based at v0:

In our example, the four edges not in **T** are d, f, h, k.
The corresponding loops (based at the basepoint v0) are
bˆcˆdb
bˆefg
aˆha
kcb

The fundamental group $\pi_1(G)$ is a free group of rank 4, and the loop classes [bˆcˆdb], [bˆefg], [aˆha], [kcb] are a free basis for the group.

[end of handout - when we want to analyze a covering space, it can help to be fussy/careful in this way]