To M201 students-

(As discussed in class yesterday) Our next topic is the so-called “Van Kampen theorem” (or “Seifert-Van Kampen theorem”). This theorem tells us how to compute the fundamental group of the union of two sets, X and Y, if we understand the fundamental groups of X, Y, and their intersection.

In order to state the theorem (the way I want to for our class), we need to learn about “group presentations”. I have posted four handouts that introduce you to this algebra topic, as well as other material on groups that we will use in the course. (The handouts were written in different semesters, but they fit together in the order they are numbered.) Your assignment for next WEDNESDAY Oct 4 is to study those handouts and for Friday Oct 6, to do several HW problems. (Remember there is no class this Monday Oct 2.)

My plan for next Wednesday is to assume you have read the handouts so we can talk comfortably in the language of group presentations. I am going to state the Van Kampen theorem in terms of presentations of the various fundamental groups.

In handout Groups II, the “text” is Massey.

Here are the HW problems due next Friday. There are so many hints, the problems should be easy for you IF you START EARLY so you have time to read carefully through the handouts.

2. Groups I page 4 last paragraph.
4. Groups II page 2, at the very end of section 0.1 There is a question, “why?” Answer that question by showing that the set S = {2,3} does not satisfy Theorem 1 at the top of that page. This shows that {2,3} is not a basis.
5. (I am writing this note using a simple text editor; so the symbol a’ denotes the inverse of element a, that is a’ denotes a^{-1}) This problem is based on Groups III.

Here are two ways to calculate the fundamental group G of the Klein bottle. First, if we construct the Klein bottle as a square with identifications, we have G1::<a,b | aba'b>. On the other hand, if we construct the Klein bottle by sewing two Mobius bands together along their boundaries, we get G2::<x,y | x^2 = y^2>. So these two presentations must give isomorphic groups. Prove G1 is isomorphic to G2. (You can use Tietze transformations, or write-and-check-that-it-works an explicit isomorphism.)

6. (This problem is based on handout Groups IV).
Let $G$ be the group $\langle a, b \mid a^2 = b^3 \rangle$. Show that the abelianization of $G$ is isomorphic to the additive group of integers, $\mathbb{Z}$.

(For the following two problems, you may assume the following “gifts” that we will prove later. You may need to study the Groups handouts in order to understand the language in the second “Gift Proposition”.)

Gift Proposition: Suppose $X$ is the union of some number of 2-spheres all meeting at one common point; that is, $X = \text{wedge}$ of some number of copies of $S^2$. Then $X$ is simply connected.

Gift Proposition: Suppose $X$ is the union of some number of 1-spheres all meeting at one common point; that is $X = \text{wedge}$ of $n$ of copies of $S^1$. Then the fundamental group of $X$ is a free group of rank $n$, with presentation $\langle a_1, \ldots, a_n \mid \rangle$ (i.e. $n$ generators, no relations).

Hint for the two following problems: In each case, find a strong deformation retract of the given space whose fundamental group you know.
