# General Topology (22M:132)
# Fall 2007

## Course Description



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| Professor | Jonathan Simon | 1-D MLH, 335-0768, jonathan-simon@uiowa.edu |
| | | Course information http://www.math.uiowa.edu/∼jsimon/ |
| TA | Amanda Fontanez | B20-J MLH, 335-3650, amanda-fontanez@uiowa.edu |



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| Class time | MWF | 10:30, 221 MLH |
| | Tues | 10:30, 118 MLH |

*Topologists study the "shapes" of sets.*
*They spend half their time defining what that means,*
*and the other half doing it.*

### Introduction

In first year Calculus, you learned these theorems:

**Theorem.** *If $f$ is a continuous function on the domain $X = [a, b]$ such that $f(a) < 0$ and $f(b) > 0$, then somewhere in $X$ there is a point $x$ where $f(x) = 0$.*

**Theorem.** *If $f$ is a continuous function on the interval $X = [a, b]$ then there is a point $x_M$ in $[a, b]$ such that $f$ attains its maximum value at $x_M$; likewise there is a point $x_m$ where $f(x_m)$ is the minimum valueof $f$ on $[a, b]$.*

The proofs of these theorems (which you probably saw in a later course) depend on "topological properties of the domain $X$ : for the first theorem, the fact that $X$ is *connected*; and for the second, that $X$ is *compact.*

What would happen if the domain $X$ is some other set?

Are the theorems correct if we allow $X$ to be the whole set of reals, $\mathbb{R}$? The unit disk $D^2 = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$? The cartesian product of two intervals $[a, b] \times [c, d] = \{(x, y)\ x \in [a, b] \text{ and } y \in [c, d]\}$? The cartesian product of *infinitely many* intervals?

When we set a computer trying to find approximate solutions to equations, we would like to know that a solution really does exist before letting the computer churn away forever. Does there exist a complex number $z$ for which $f(z) = z^6 - 13z^4 + z^2 + z + 200 = 0$?

What if $X$ is a set whose *elements* are themselves functions? For example, $X = \{f : [0, 1] \to \mathbb{R} \mid f \text{ is continuous }\}$. What would it mean to say that some function $F : X \to \mathbb{R}$ is "continuous"?

There are many reasons to study topological ideas. Here is just one example: Suppose you are studying some physical system. You measure 17 parameters in the system, and model each state of the system as the list of parameters, so a point in $\mathbb{R}^{17}$. The system satisfies some constraints; these are equations that the coordinates must satisfy; so the physically feasible state-space is a subset of $\mathbb{R}^{17}$. On this state-space, you have defined some notion of energy. The question is whether there exists a minimum-energy feasible state. If the state-space has the right topological properties, and the energy function is continuous, then there is an optimal state. In differential topology and algebraic topology (maybe a little introduction in M132 if we have enough time) you will develop more ideas about "shapes of sets" and be able to prove more subtle theorems such as (for a given feasible domain) a lower bound on how many critical states must exist *regardless of the definition of the energy function* so long as the "energy" is a continous function.

*The topological* **"shape"** *of the domain of a [continuous] function can force the behavior of the function*, such as existence of maxima and minima or existence of solutions of equations

In this course we start with basic ideas of topology, notions of "proximity" or "convergence". If $X$ and $Y$ are some sets on which we have some understanding of proximity or convergence, then functions from $X \to Y$ which respect these are called "continuous". We can define useful properties of spaces, and mappings between them, in terms of these core ideas. For example, there are several ways to describe "how big" is a space, or "how many pieces" it has, or when two spaces are essentially the same even if their original definitions look different. As you have probably seen before, (e.g. in a course like our 22M:55 or 22M:130), an open interval $(a, b)$ in the real line $\mathbb{R}^1$ is *topologically equivalent* to the open ray $(0, \infty)$, but topologically different from either a closed interval $[a, b]$ or the union of separated intervals $X = [a, b) \cup [b + 1, c)$ . And you may already have studied such ideas in the context of spaces such as the the plane $\mathbb{R}^2$, or subsets of the line or plane. Our goal now is to take these ideas to a higher level of generalization where the spaces we consider might be familiar sets with different notions of distance, or sets in higher dimensional space $\mathbb{R}^n$ or spaces whose "points" are themselves mappings between other sets.

Here are two questions just to start you thinking. By the end of the course, you should be able to settle the first, and suggest several answers to the second.

(1) From algebra, you know the definition of a group. A *topological group* is a topological space $X$ that also is a group, such that the group operations of multiplication (as a function from $X \times X \to X$) and inversion ($x \to x^{-1}$, as a map from $X$ to $X$) are continuous. An easy example is the set $\mathbb{R}^1$ with the usual notion of continuity and the group operation of usual addition. Can you give a closed interval $[a, b]$ (with the usual notion of continuity) the

structure of a topological group by defining some novel group operation (why does usual addition not make sense here)? Since $(0,1)$ is topologically equivalent to $\mathbb{R}^1$, we **can** define an operation on $(0,1)$ making it a topological group. How do you define "addition"??

(2) Let $X$ be the set of continuous (in the usual sense) functions from $\mathbb{R}^1$ to $\mathbb{R}^1$. In what sense can we say that a sequence of elements $(f_n)_{n\in\ Z}$ of $X$ "converges" to a particular element $f$?

**Textbook.** Our text is *Topology (Second Edition)* by J. Munkres.

We will cover the following sections, plus additional material as time permits. The meaning of "cover" will vary; some sections we will study in depth; others we will only briefly discuss. I will make clear during the course which material you are responsible for on exams and which material you can browse just to be an educated mathematical citizen.

[Note: Many of you, are taking 22M:115 or have already taken it. We may skip or treat only lightly some sections of our text because that is/was covered in M115. ]

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| Chapter 1 | Quick review Sec. 1-7; skip 8, do Sec. 9,10 later ; skip Sec 11. |
| Chapter 2 | all |
| Chapter 3 | all |
| Chapter 4 | all (less on metrization, more on Tietze); |
| Chapter 5 | Sec 37, skip 38 |
| Chapter 6 | skip (but you should read Section 41 for future courses) |
| Chapter 7 | all, but avoid duplication with with M115 |
| Chapter 8 | all, but avoid duplication with with M115 |
| Chapters 9, 10, 12 | Some parts, if time permits. |

**TA Sessions.** Our class meets 4 days/week. Usually Prof. Simon will meet with you on MWF and TA Amanda Fontanez will work with you on Tuesdays; on rare occasions we might switch. In addition to grading Homework (see below) and meeting with the whole class for "Discussion" on Tuesdays, the TA also will have times to work individually with students who are having difficulties with some of the material. The professor also will have office hours and students are welcome to seek help then too.

**EXAMS.** There will be two **evening** midterm exams, 7-8:30 p.m., on approximately October 1 and November 4. Dates will be confirmed well ahead of time. I will try to design a one-hour exam and then give you an hour-and-a-half to do it.

The FINAL EXAM is Thursday December 20 7:30-9:30 a.m. in our regular classroom 221 MLH.

**Special dates.** Monday Sept. 3 – no class
Friday September 14 – no class

**Homework.** I expect to assign homework each class, with assignments collected each Wednesday. **Un**excused late homework is eligible for half credit if it is handed in within one week of the due-date. Of course, if there is an illness or other personal emergency, we can make special arrangements. I encourage you to study in groups in order to master the course material; but you should not do "joint homework": Be sure the homework you hand in is actually your own work. (See University policy on plagiarism listed below.)

**Grading.** Your work on exams and homework will be averaged with the following weights:

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| MIDTERM I | 20 % |
| MIDTERM II | 20 % |
| FINAL EXAM | 30 % |
| HOMEWORK | 30 % |

Students' final grades **may** be above the strict average; some of the reasons in the past that I have used for such bonuses have been: excellent class participation, improvement through the semester, or some especially impressive piece of work. I do often take attendance, and include regular attendance as part of "class participation".

**Special Notes.** I expect that we will enjoy each others' company and efforts during this course, and that we will deal with each other and with the course work in an honorable professional way. But sometimes in life there are disagreements or problems; if something arises that we cannot settle among ourselves, you may wish to contact the Mathematics Department Chair. His office is in 14MLH; to make an appointment, call 335-0714 or contact the Department Secretary in 14 MLH. You also are welcome to tell the Chair good things. Also, please let me know if you have a disability which requires special arrangements; students with such disabilities should be in contact with the appropriate University office, and I will be happy to help you make this connection if you have not already done so.

**Universal Disclaimer.** This Course Description represents my intentions and best current estimate of details. Changes may be announced in class and/or by email.

**Additional material required by the College of Liberal Arts and Sciences.**

*Academic Fraud.* Plagiarism and any other activities that result in a student presenting work that is not his or her own are academic fraud. Academic fraud is reported to the departmental DEO and then to the Associate Dean for Academic Programs and Services in the College of Liberal Arts and Sciences.
www.clas.uiowa.edu/students/academic_handbook/ix.shtml

*Making a Suggestion or a Complaint.* Students have the right to make suggestions or complaints and should first visit with the instructor, then with the course supervisor if appropriate and next with the departmental DEO. All complaints must be made within six months of the incident.
www.clas.uiowa.edu/students/academic_handbook/ix.shtml#5

*Accommodations for Disabilities.* A student seeking academic accommodations first must register with Student Disability Services and then meet with a SDS counselor who determines eligibility for services. A student approved for accommodations should meet privately with the course instructor to arrange particular accommodations. See www.uiowa.edu/~sds/

*Understanding Sexual Harassment.* Sexual harassment subverts the mission of the University and threatens the well-being of students, faculty, and staff. See www.sexualharassment.uiowa.edu/ for definitions, assistance, and the full policy.

*Administrative Home of the Course.* The administrative home of this course is the College of Liberal Arts and Sciences, which governs academic matters relating to the course such as the add / drop deadlines, the second-grade-only option, issues concerning academic fraud or academic probation, and how credits are applied for various CLAS requirements. Please keep in mind that different colleges might have different policies. If you have questions about these or other CLAS policies, visit your academic advisor or 120 Schaeffer Hall and speak with the staff. The CLAS Academic Handbook is another useful source of information on CLAS academic policy. www.clas.uiowa.edu/students/academic_handbook/index.shtml

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