Exercise 24.9. Let \( A \subseteq \mathbb{R}^2 \) be countable. Then \( \mathbb{R}^2 - A \) is path connected.

**Proof.** Let \( x, y \in \mathbb{R}^2 - A \). Since there are only countably many points in \( A \), there are only countably many lines through \( x \) that hit \( A \). Similarly, there are only countably many lines through \( y \) that hit \( A \). Therefore there are at least two lines through \( x \) that do not hit \( A \) and one line through \( y \) that does not hit \( A \). (Actually there are uncountably many, but this is all we need.) Then the line through \( y \) must intersect one of those two lines through \( x \) in a point \( z \). We can then form a path from \( x \) to \( y \) in \( \mathbb{R}^2 - A \) by taking the union of the line segment from \( x \) to \( z \) with the line segment from \( z \) to \( y \) and appropriately parameterizing this geometric construction to form a continuous function \( f : [0, 1] \rightarrow \mathbb{R}^2 \) with \( f(0) = x \) and \( f(1) = y \). \( \Box \)