Exercise 24.9. *Let $A \subseteq \mathbb{R}^2$ be countable. Then $\mathbb{R}^2 - A$ is path connected.*

Proof. Let $x, y \in \mathbb{R}^2 - A$. Since there are only countably many points in A, there are only countably many lines through x that hit A. Similarly, there are only countably many lines through y that hit A. Therefore there are at least two lines through x that do not hit A and one line through y that does not hit A. (Actually there are uncountably many, but this is all we need.) Then the line through y must intersect one of those two lines through x in a point z. We can then form a path from x to y in $\mathbb{R}^2 - A$ by taking the union of the line segment from x to z with the line segment from z to y and appropriately parameterizing this geometric construction to form a continuous function $f : [0, 1] \to \mathbb{R}^2$ with $f(0) = x$ and $f(1) = y$. □