More on graphing surfaces....

> with(plots):

Here are more examples of how you can analyze and visualize a function of several variables (in
particular, a function of the form
\[ z = f(x, y) \].

Example 1.

\[ f := x^2 + y \]

(1)

We want to understand/see the graph of \( f \), that is all points of the form \([x, y, x^2 + y]\) in \( \mathbb{R}^3 \).

(Please note - when I am using Maple to write notes for you, I'd like to denote points with square
brackets such as [1,2,3] rather than with parentheses (1,2,3). This is because Maple undersands [1,2,3]
but not (1,2,3). When in Maple-land, I'll try to talk Maple-ese.

Step 1. Are there any symmetries?

The formula for \( f \) only involves \( x^2 \), so if a point \((x, y, z)\) is on the graph, so is the point \((-x, y, z)\). This
means the graph is symmetric to the yz-plane.

Step 2. Vertical slices with fixed \( x \) and varying \( y \):

If we fix \( x \), e.g. \( x = 3 \), the function becomes \( f(3, y) = 9 + y \). This is a linear function of \( y \). If we use a
different value of \( x \), we'll get a line with the same slope (slope=1 here), just higher or lower. So the
vertical slices of the graph gotten by fixing \( x \) (and letting \( y \) very) are lines of slope 1. The heights of
these lines change as \( x \) changes.

> L1:=tubeplot([0, y, 0^2+y], y=-2..2, radius=.1):
> L2:=tubeplot([1, y, 1^2+y], y=-2..2, radius=.1):
> L3:=tubeplot([-1, y, (-1)^2+y], y=-2..2, radius=.1):
> L4:=tubeplot([2, y, 2^2+y], y=-2..2, radius=.1):
> L5:=tubeplot([-2, y, (-2)^2+y], y=-2..2, radius=.1):
> L6:=tubeplot([3, y, 3^2+y], y=-2..2, radius=.1):
> L7:=tubeplot([-3, y, (-3)^2+y], y=-2..2, radius=.1):
> display({L1, L2, L3, L4, L5, L6, L7}, axes=boxed, labels=[X, Y, Z],
          orientation=[-49,70]);
Step 3. Vertical slices with fixed $y$ and varying $x$:
If we fix $y$, e.g. $y=3$, the function becomes $f(x,3) = x^2+3$. This is a standard parabola with vertex at height 3. If we use a different value of $y$, we'll get the same shaped parabola, just translated up or down some amount. The vertical slices of the graph gotten by fixing $y$ (and letting $x$ vary) are parabolas whose heights change with $y$. When $y$ is big, the parabolas' vertices are high up; when $y$ is negative, the parabolas' vertices are below the xy-plane.

```plaintext
> P1:=tubeplot([x, 0, x^2+0], x=-2..2, radius=.05):
> P2:=tubeplot([x, 1, x^2+1], x=-2..2, radius=.05):
> P3:=tubeplot([x, -1, x^2-1], x=-2..2, radius=.05):
> P4:=tubeplot([x, 2, x^2+2], x=-2..2, radius=.05):
> P5:=tubeplot([x, -2, x^2-2], x=-2..2, radius=.05):
> P6:=tubeplot([x, 3, x^2+3], x=-2..2, radius=.05):
> P7:=tubeplot([x, -3, x^2-3], x=-2..2, radius=.05):
> display({P1, P2, P3, P4, P5, P6, P7}, axes=boxed, labels=[X, Y, Z], orientation=[-49,70]);
```
If you look at the two sets of slice-curves (especially if you combine them), you might already feel confident about what the graph of the function looks like. Let's ask the computer to draw the graph.

> plot3d([x, y, f], x=-3..3, y=-3..3, axes=boxed, labels=[X,Y,Z], orientation=[-49,70]);
Just as a further check, and also to practice with "level curves", let's see what level curves and contours look like for this function.

> C1:=implicitplot(f=0, x=-3..3, y=-3..3, color=blue): display(C1);

> C2:=implicitplot(f=1, x=-3..3, y=-3..3, color=magenta):
> C3:=implicitplot(f=2, x=-3..3, y=-3..3, color=red):
> display({C1, C2, C3});
It is a little hard to draw these level curves on the same picture as the graph of \( f \) and/or to show the contours "by hand" in Maple. But Maple has a built-in option for plotting surfaces that will show the contours.

```maple
> plot3d([x,y,f], x=-3..3, y=-3..3, axes=boxed, labels=[X,Y,Z],
orientation=[-49,81], style=patchcontour);
```

Just to emphasize that the contours are the parts of the graph surface that lie at fixed heights, I will have the computer draw the graph surface, and also the plane at height 3, so you can see that the contour curve IS the intersection of the graph with that plane.

```maple
> DrawGraph:=plot3d([x,y,f], x=-3..3, y=-3..3, axes=boxed, labels=
[X,Y,Z], orientation=[-49,81], style=patchcontour):
> DrawHorizPlane:=plot3d([x,y,3],x=-3..3, y=-3..3, axes=boxed,
labels=[X,Y,Z], orientation=[-49,81], style=patchnogrid, color=grey):
```
If we just want to draw the level-curves in the domain, that is in the $xy$-plane (see curves C1, C2, C3 above), Maple has a built-in operation to do that. In case you "forget" the syntax for some command in Maple, you can get online help by entering "?command", to get help on "command". The help comes in a different window, which you can minimize or quit without losing your main worksheet.

```maple
> display({DrawGraph, DrawHorizPlane});
```

In the above version, I have no idea what values of $f$ are represented by the curves. We can (at least partly) solve that communication problem by telling Maple which level values to use. So here are the level curves $f(x,y)=1$, $f(x,y)=2$, $f(x,y)=3$, $f(x,y)=-1$, $f(x,y)=-2$, $f(x,y)=-3$.

```maple
> contourplot(f, x=-3..3, y=-3..3);
```

```maple
> contourplot(f, x=-3..3, y=-3..3, contours = [0, -1, 1, -2, 2, -3, 3]);
```
The contour curves are not labeled with their values, but now that we know what the values are, we can figure out which curves correspond to which values.

END OF HANDOUT