The functions $z = x^2 + y^2$ and $z = \sqrt{x^2 + y^2}$ both have level-curves that are concentric circles. But the curves are distributed in different ways.

> with(plots):

Warning, the name changecoords has been redefined

> Graph1:=plot3d([x,y,x^2+y^2], x=-3..3, y=-3..3, style=patchcontour):

> Contours1:=tubeplot({[cos(t), sin(t), 0], [sqrt(2)*cos(t), sqrt(2)*sin(t), 0], [sqrt(3)*cos(t), sqrt(3)*sin(t), 0], [sqrt(4)*cos(t), sqrt(4)*sin(t), 0]}, t=0..2*Pi, radius=.05, color=black):

> Contours1Plain:=contourplot(x^2+y^2, x=-3..3, y=-3..3, contours=[1,2,3,4]):

> display({Graph1, Contours1});
Notice that the level-curves are packed closer together as the circles get larger. This tells us that the function $z=x^2+y^2$ is growing at a faster rate (with respect to $||(x,y)||$) as the points $(x,y)$ get farther from the origin.

Compare this to what you studied in Calc I: The derivative of $f(x)=x^2$ is $f'(x) = 2x$. The *rate* of growth itself grows as $x$ grows.

Now let's see what the level curves look like for $f(x)=\sqrt{x^2+y^2}$.

```maple
> Graph2:=plot3d([x,y,sqrt(x^2+y^2)], x=-3..3, y=-3..3, style=patchcontour):
> Contours2:=tubeplot({[cos(t), sin(t), 0], [(2)*cos(t), (2)*sin(t), 0],
> [(3)*cos(t), (3)*sin(t), 0], [(4)*cos(t), (4)*sin(t), 0]}, t=0..2*Pi,
> radius=.05, color=black):
> Contours1Plain:=contourplot(sqrt(x^2+y^2), x=-3..3, y=-3..3,
> contours=[1,2,3,4]):
> display({Graph2, Contours2});
```
Without the surface, just looking at the level-curves, we would see

> contourplot(sqrt(x^2+y^2), x=-3..3, y=-3..3, contours=[1,2,3,4], color=black, scaling=constrained);
Notice that the level-curves are packed at constant distance from each other as the circles get larger. This tells us that the function \( z = \sqrt{x^2 + y^2} \) is growing at a constant rate (with respect to \( \| (x,y) \| \) ) regardless of how far the points \((x,y)\) are from the origin.

This is consistent with what you learned in Calc I: For the function \( f(x) = |x| \), the derivative (for \( x > 0 \)) is constant: \( f'(x) = 1 \).

NOTE: These observations about how closely packed are the level-curves only makes sense if we are using level curves corresponding to values of the function that are a constant amount apart (as in the above, where we plotted the level-curves corresponding to \( z = 1, z = 2, z = 3, z = 4 \)). If we had plotted curves corresponding to \( z \) values that were different distances apart, it would be hard to learn about the behavior of the functions.

MORAL OF THE STORY:
If you draw level-curves for a function \( z = f(x,y) \) corresponding to values of \( z \) that are a constant distance apart, then close packed level-curves \( \implies \) rapidly changing function, and widely spread out level curves \( \implies \) slowly changing function.

This is an important principle for you to learn (for our course and "outside"). Sometimes, the only way a function is communicated to us is via level-sets, so we want to be able to learn properties of the function just by looking at the level-sets.

In the next handout(s), you can see examples of this principle in action.

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end of handout
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