1. (12 pts.) Let \( \mathbf{u} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k} \) and \( \mathbf{w} = \mathbf{i} + \mathbf{j} + 3\mathbf{k} \).

(a) \( \mathbf{u} \cdot \mathbf{w} = \) 

(b) \( \mathbf{u} \times \mathbf{w} = \) 

(c) \( ||\mathbf{w}|| = \) 

(d) the unit vector that points opposite the direction of \( \mathbf{w} = \) 

(e) Which one, or more, of the following equals zero?

\[ ||\mathbf{u} \times \mathbf{u}|| \quad \mathbf{u} \cdot \mathbf{u} - ||\mathbf{u}|| \quad \mathbf{u} \cdot (\mathbf{w} \times \mathbf{u}) \]

(f) Give a vector perpendicular to \( \mathbf{u} \)
A plane is flying (i.e. aiming) in direction 30° north of east (see diagram below) with an airspeed of 250 miles/hour. But the air in which the plane is flying is moving (i.e. the wind) directly east at 60 miles/hr. What is the plane's actual velocity relative to the ground?

CHANGE FOR 2005 --- REPLACE THIS WITH A PROBLEM ABOUT THE CHAINRULE, SUCH AS: If \( f(x,y) = (x \cos(y), x \sin(y)) \) and \( a \) is the point \((1,2)\), then \( Df(a) = \)__________

Match each equation with its graph. The equations are on this page, the graphs are on the next page. Mark each graph with its appropriate equation.

(Note: All the graphs are drawn with the x- and y- axes roughly horizontal and the z-axis pointing approximately "up". But the axes for the various graphs may have been rotated relative to those for other graphs, just to make the pictures easy to see. In particular, you should not assume that the x-axis and y-axis are the same from one picture to the other.

a) \( z = (2/10) x - (3/10) y \)
b) \( z = x \sin(y) \)
c) \( z = x^2 + y^2 \)
d) \( z = (x^2 + y^2 + 1)^{-1} \)
e) \( z = \sin(x y) \)
f) \( z = x^3 + 2 \)
g) \( z = x^2 - y^2 \)
4. (10 pts) Sketch three level curves for each of the following functions:

a) \( f(x,y) = x^2 - 2y \).

b) \( f(x,y) = \sin(x+y) \).

5. (5 pts) Describe the level sets for the function

\[ f(x,y,z) = \frac{z}{\cos(x^2 + y^2)^4} \].

CHANGE FOR 2005: REPLACE THIS WITH:

If \( f(x,y) = \cos(x^2+y) \), find all second partial derivatives of \( f \).
For the function $f(x,y) = xy - x^3y$, find the following:

a) The partial derivative of $f$ with respect to $x$ at the point $(1,2)$.

b) The partial derivative of $f$ with respect to $y$ at the point $(1,2)$.

c) The gradient of $f$ at the point $(1,2)$.

d) The directional derivative of $f$ in the direction of the vector $3\hat{i} + 7\hat{j}$ at the point $(1,2)$.

e) The largest possible value for the $D_u f(1,2)$, where $u$ ranges over all unit vectors.

f) A direction vector (i.e. unit vector) $u$ for which $D_u f(1,2)$ achieves the preceding maximum value.

g) A direction vector (i.e. unit vector) $w$ for which $D_w f(1,2) = 0$. 
7. (8 pts) Here is a table of values of some function \( f(x,y) \).

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</table>

Use (some of) this data to estimate \( \nabla f(1.1, 4) \).

8. (8 pts) The three points \((0,0,0), (2,3,4), \) and \((0,0,1)\) determine a plane in 3-space. Find a vector that is perpendicular to this plane.

9. (9 pts) Find an equation for the plane that is tangent to the surface \( xy + x^2 + xz = 6 \) at the point \((1,2,3)\).

ADD FOR 2005

10. (?? pts) If \( f(x,y) = ((x-y)^{10}/\cos(1+(x-y)^3))^*\sqrt{1+(x-y)^4}) \), show that the partial derivatives \( df/dx \) and \( df/dy \) satisfy a certain partial differential equation.