To M28 students -
Hello again. One of the students in the class asked for help with some of the HW. May as well share the discussion with everyone...

On Tue, 1 Feb 2005 [a student] wrote:

> Hi, I had a couple of questions on pg. 28 #20 and pg. 51 #8 and #16. I was little confused on those. If you could help me on those problems, that would be great.
>
> Thanks
>
> Here are some hints to get you started. Of course, my goal is for you to really understand what you're doing, so these "hints" will include general discussions as well as particular approaches to given problems.

(Since there's no boldface in this plain text email, I will write vectors as CAPital letters)

Page 28 #20
First, visualize the situation...You have some kind of ramp (like Figure 1.38 on page 23); the slanted face of the ramp "aims" in the direction of vector 4I+J.

(Remark: In Fig 1.38, the ramp tilts up at a 30 degree angle. So the slanted face is parallel to [any] vector [that's a positive scalar multiple of] (cos 30 deg)I + (sin 30 deg) J = (1/2)sqrt(3) I + (1/2) J . We also could use, e.g., sqrt(3)I + J or any other positive multiple having the same direction. But in Problem 20, we are *given* the vector explicitly, so don't need to do any trig to find that direction.)

Back to the problem...
The vector force F = I-2J is not straight down (in contrast to gravity in Fig 1.38), but aims in some other direction. So, if you want to continue
having a simple physical situation to imagine, you'll have to imagine that the force is due to some combination of (for example) gravity pulling down and a "wind" pushing sideways. Or make up your own scenario. The math is the same.

We want to write $F = F_1 + F_2$, where $F_1$ is a vector parallel to the given direction of motion, that is $F_1$ is parallel to $A=4I+J$, and $F_2$ is perpendicular to vector $A$. (I'm writing "A" instead of "a" to remind us it's a vector.)

Example 4 on page 25 and/or your class notes will remind you how to find the component of $F$ in the direction of vector $A$.

You can do more work to find a normal vector to $A$ and then project $F$ onto that direction. BUT BUT BUT Here's a neat trick:

Theorem: If $F = F_1 + F_2$, and $F_1$ is THE projection of $F$ onto $A$, then $F_2$ is perpendicular to vector $A$. That's a nice intuitive idea of what projection does: it extracts from $F$ as much as possible of $F$ in the direction of $A$. What's left has no "A left in it".

Proof: Calculate the projection of $F_2 (= F-F_1)$ onto $A$ and see you get 0.

So once you find $F_1$, you can take $F_2$ to be just $F-F_1$.

BUT BUT BUT you should check the result just to make sure. That is, calculate $F_2=F-F_1$ and make sure it really is perpendicular (via the dot product test) to $A$.

Section 1.5

Before discussing the particular problems, let's recall some basics: (1) Given a normal ("joystick") vector $N = aI+bJ+cK$ to a plane, and a point $(1,2,3)$ on the plane, then the plane has equation $a(x-1)+b(y-2)+c(z-3)=0$ (or $ax+by+cz = (a+b+c)$). CONVERSELY (Read Example 2 page 44), given a plane with equation $ax+by+cz=d$, the vector $aI+bJ+cK *is* normal to the plane.

(Note: this is easy to check: Suppose $(x_1,y_1,z_1)$ and $(x_2,y_2,z_2)$ are two points on the plane. Then $ax_1+by_1+cz_1 = d = ax_2+by_2+cz_2$, so $a(x_2-x_1) + b(y_2-y_1) + c(z_2-z_1) = 0$. That is, the vector $aI+bJ+cK$ dotted with the vector $(x_2-x_1)I+(y_2-y_1)J+(z_2-z_1)K = 0$. This is true for all pairs of points in the plane satisfying the given equation, so all lines in the plane are perpendicular to vector $aI+bJ+cK$.)

(2) Given two vectors in (rather, parallel to) the plane, we can take
their cross product to get a vector normal to the plane.

OK, now for the particular problems...

Page 51 #8
If you are given a parametric equations for a line, you know the direction of the line.

For example, the line \( x=2t+3, \ y=4t+5, \ z=6t+7 \) is the line through point \((3,5,7)\) in the direction of vector \(2\mathbf{I}+4\mathbf{J}+6\mathbf{K}\). And if you are given parametric equations for a line, you certainly can find a point on the line. Now you have "standard" data from which to write an equation for the plane.

Page 51 #16
This we did just at the end of class on Monday. I worked an example just like this in the handout. See also Example 4 (second method), bottom page 45 - top page 46.

I hope these comments are helpful.

Sincerely,
Jonathan Simon