Examples of [indefinite] integration
i.e.
antidifferentiation

Here are three examples of the "method of substitution". I hope this also (see Example 4) will make clear the syntax for using Maple to evaluate indefinite integrals.

EXAMPLE 1.

\[ f := x^2 \cos(x^3); \]
\[ \int x^2 \cos(x^3) \, dx \]
\[ = \frac{1}{3} \sin(x^3) \]

This is a standard example of integration via the method called "substitution" (Section 6.2).
A person, presented with the integration problem, sees that the function \( f \) to be integrated is actually of the form \( (\text{do something to an expression in } x) \cdot (\text{the derivative of that expression}) \).

Let \( u = x^3 \). Then \( du/dx = 3x^2 \), i.e.
\[ \frac{du}{dx} = (3x^2) \, dx \], i.e.
\[ \frac{1}{3} \, du = x^2 \, dx. \]

So the integral
\[ \int x^2 \cos(x^3) \, dx \]
can be rewritten in terms of \( u \), by substituting:
\[ \cos(x^3) = \cos(u), \quad x^2 \, dx = \frac{1}{3} \, du; \]
\[ \frac{1}{3} \cos(u) \, du \]
and that we know is

\[ (1/3) \sin(u) = (1/3) \sin(x^3); \]

\[ \frac{1}{3} \sin(u) = \frac{1}{3} \sin(x^3) \]

[I am leaving off the +C that should be included with antiderivatives.]

EXAMPLE 2 (6.2 page 285 number 3)

\[ \int \frac{1}{2y-1} \, dy \]

We see the function is of the form \((1/\{\text{linear expression in } y\})\). Any time you are dealing with an integrand (that's the fancy name for the function being integrated) that is a function OF a linear expression in the variable, the problem will get simpler if you rewrite the integral in terms of that linear expression as the new variable. [If the expression is NON-linear, then you need to see the "du/dvariable" as a separate expression in order for the substitution to have this guarantee of making life simpler.]

Here let

\[ u=2y-1; \]

\[ u = 2y - 1 \]

\[ \frac{du}{dy} = 2 \implies du = 2 \, dy \implies dy = \frac{1}{2} \, du. \]

\[ \int \frac{1}{2y-1} \, dy = \int \frac{1}{u} \, du \]

\[ \frac{1}{2} \int \frac{1}{u} \, du = \frac{1}{2} \ln(u) \]

\[ \frac{1}{2} \ln(u) = \frac{1}{2} \ln(2y-1) \]

We could also write this as \(\ln(\sqrt{2y-1})\).

\[ \frac{1}{2} \ln(2y-1) = \ln(\sqrt{2y-1}) \]
EXAMPLE 3 (page 286 #23)

\[ f(x) := \frac{1}{x(1+\ln(x))} \]

We see this is of the form \((\text{function of } \ln(x)) \times (\text{derivative of } \ln(x))\).

We can calculate the integral, via substitution, letting \(u = 1+\ln(x)\).

Let \(u = 1+\ln(x)\). Then the integrand is \(\frac{1}{u} \times (\text{derivative of } u)\).

That is, \(\frac{du}{dx} = \frac{1}{x}\), so \(du = \frac{1}{x}dx\). Thus

\[
\int f(x) \, dx = \int \frac{1}{u} \, du
\]

Now substitute back...
\[
\int 1/u \, du = \ln(u)
\]

and \(\ln(1+\ln(x))\) is our ultimate answer.

NOTE: The key to success with "substitution" is to be so good at calculating simple derivatives that you SEE the pattern in a given problem.

You DO have to be able to do simple substitution integrals for calculus quizzes and exams. The justification for me is that "substitution" is a constant review of the ChainRule, which is an important principle deserving of review. Also scientists do sometimes want to change the variable(s) they are using to model some situation, so the general process of changing variables is worth studying.

For the specific task of evaluating indefinite integrals, all the knowledge in Sections 6.2, 6.3, 6.4, 6.5, and 6.6 has been taught to computers. You have a few homework problems where you are asked to use the computer (e.g. Maple) to calculate antiderivatives, just to make sure you will have the tool available for possible future use. Here is a sample, so you can see exactly the syntax that works. I will work the same example several times, since there is more than one correct syntax.

The example will be page 306 # 17. [Note - this is not just a "substitution" problem - you would need to use "integration by parts", which is section 6.6 that we are not covering at all.]

Approach 1: SLOW AND CAREFUL
> \(f := x^2 \sin(2x)\);

\[ f := x^2 \sin(2x) \]

> \(\text{Int}(f, x)\);

\[ \int x^2 \sin(2x) \, dx \]

> \(\text{value(\%)};\)

\[- \frac{1}{2} x^2 \cos(2x) + \frac{1}{4} \cos(2x) + \frac{1}{2} x \sin(2x)\]

Approach 2: SOMEWHAT MORE TERSE (and my suggestion for a good compromise between being too careful and being dangerously glib).

> \(f := x^2 \sin(2x)\);

\[ f := x^2 \sin(2x) \]

> \(\text{int}(f, x)\);

\[- \frac{1}{2} x^2 \cos(2x) + \frac{1}{4} \cos(2x) + \frac{1}{2} x \sin(2x)\]

Approach 3: DANGEROUS since you don't include a display of the function being integrated, so have no check on the accuracy of your data entry.

> \(\text{int}(x^2 \sin(2x), x)\);

\[- \frac{1}{2} x^2 \cos(2x) + \frac{1}{4} \cos(2x) + \frac{1}{2} x \sin(2x)\]

NOTE: When we enter "Int(...)" [note upper case I], the computer writes the integral but doesn't do any calculating. If we want to tell the computer to go ahead and evaluate the integral, we use the command value(\%).

The percent sign just means "the last thing the computer typed".

I like Approach 2 perhaps the best. We still make sure we have the right function \(f\). Then we use the command \(\text{int}(\ldots)\) with lower-case "i". With lower case "i", the computer sets up and evaluates the integral in one step - we don't have to say "value(\ldots)".

One final, bonus example, page 307 # 17

> \(f := (25t^2 - 9)^{(3/2)}\);

\[ f := (25t^2 - 9) \]

> \(\text{int}(f, t)\);
\[
\frac{1}{4} t (25 t^2 - 9)^{(3/2)} - \frac{27}{8} t \sqrt{25 t^2 - 9} + \frac{243}{200} \ln \left( t \sqrt{25 + \sqrt{25 t^2 - 9}} \right) \sqrt{25}
\]

The moral of this last story is twofold: (a) a person **could** evaluate the integral of \( f \), and (b) it's nice that we have computers available for when the problems get hairy.

####################end of handout ####################