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**Bratteli, Ola; Jorgensen, Palle****Wavelets through a looking glass. The world of the spectrum.** (English)

Applied and Numerical Harmonic Analysis. Basel: Birkhäuser. xxii, 398 p. EUR 85.98/net; sFr. 136.00 (2002). [ISBN 0-8176-4280-3/hbk]

This is a quite unusual wavelet book, with a fresh view on the subject. While the vast majority of wavelet books concentrate on multiresolution analysis and its applications, this book treats the topic from an operator theoretic point of view, with focus on techniques having a geometric or spectral theoretic flavor. In fact, while parts of the book could be used in a wavelet course, other parts would be suitable in a course directed towards operator theory. Each chapter (and some of the sections) starts with an informal tutorial, which explains the main ideas in less technical terms. Furthermore chapter 1 contains a section with terminology, which explains the main concepts and the meaning of the key words in e.g., engineering, physics, and mathematics. Both features certainly helps the reader. Each chapter concludes with exercises of varying difficulty. Chapter 1 contains introductory material, like Fourier series, multiresolution analysis, and the cascade algorithm. The transfer operator, one of the central topics in the book, is introduced: given a trigonometric polynomial  $m_0$  and a positive integer  $N$ , it is defined on certain functions  $f$  on the torus  $T$  by

$$R_{m_0}f(z) = \frac{1}{N} \sum_{w \in T, w^N = z} |m_0(w)|^2 f(w).$$

A large part of the book concerns this operator, e.g., analysis of properties of  $R_{m_0}$  on certain function spaces based on assumptions on  $m_0$ . Chapter 2 deals with homotopy theory and convergence properties of the cascade algorithm, which turns out to be related with the spectrum of a certain transfer operator. The chapter concludes with some open problems concerning vector-valued versions of the scaling identity. Chapters 3 and 4 deal with a more general definition of transfer operators than stated above (Ruelle operators) and discuss their spectral properties, e.g., properties of the restriction of the transfer operator to  $C(T)$ . Chapter 6 demonstrates how biorthogonal wavelets might be turned into orthogonal wavelets in a fashion which preserves the essential spectral properties (isospectral orthonormalization).

*Ole Christensen (Lyngby)**Keywords*: wavelets; cascade approximation; cycle; homotopy; transfer operator; Ruelle operator;  $C^*$ -algebra*Classification* :

- \*42-02 Research monographs (Fourier analysis)
- 42C40 Wavelets
- 43A65 Representations of groups, etc. (abstract harmonic analysis)
- 47Nxx Appl. of operator theory
- 81Pxx Axiomatics, foundations, philosophy of quantum theory
- 94A12 Signal theory

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