Jorgensen, Palle E. T. (1-IA); Ólafsson, Gestur (1-LAS)

Unitary representations of Lie groups with reflection symmetry. (English. English summary)


Let $G$ be a real Lie group with a nontrivial involutive automorphism $\tau$. A unitary representation $\pi$ of $G$ on a Hilbert space $H$ is said to be reflection symmetric if there exists an involutive unitary operator $J$ on $H$ such that $\pi(\tau(g)) = J\pi(g)J$, $g \in G$. Denote $H = G^\tau$. The Lie algebra $\mathfrak{g}$ of $G$ admits the decomposition $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{q}$, where $\mathfrak{q}$ is the eigenspace of $d\tau$ corresponding to $-1$. Assume a closed $H$-invariant convex cone $C \subset \mathfrak{q}$ to be given such that: $C^\circ \neq \emptyset$; $adY$ is $\mathbb{R}$-semisimple for all $Y \in C^\circ$; $S(C) = H \exp C$ is a closed semigroup, and $H \exp C^\circ$ is diffeomorphic to $H \times C^\circ$; there is a closed $S(C)$-invariant subspace $0 \neq K_0 \subset H$ satisfying $\langle v | J(v) \rangle \geq 0$ for all $v \in K_0$.

Let $G^\circ$ denote the simply connected Lie group with the Lie algebra $\mathfrak{g}^\circ = \mathfrak{h} \oplus i\mathfrak{q}$. The main result of the paper is a construction of a unitary representation $\pi^\circ$ of $G^\circ$ acting on a quotient of $K_0$ such that $d\pi^\circ(X)$ is induced by $d\pi(X)$ for $X \in \mathfrak{h}$ and $id\pi^\circ(Y)$ is induced by $d\pi(iY)$ for $Y \in C$. This construction applies to the case when $G$ is semisimple and $G/H$ is a non-compactly causal symmetric space and, in particular, a Cayley-type space, the representation $\pi^\circ$ being an irreducible unitary highest weight representation. Finally, the semidirect products $G = HN$, where $N$ is normal and abelian, are considered.

A. L. Onishchik (RS-YAR)