Endomorphisms of $\mathcal{B}(\mathcal{H})$. (English. English summary)


Let $\mathcal{B}(\mathcal{H})$ denote the algebra of all bounded linear operators on a separable, infinite-dimensional, complex Hilbert space $H$. If $\alpha: \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$ is a unital endomorphism, the authors say that $\alpha$ is ergodic [resp., a shift] if $\{T \in \mathcal{B}(\mathcal{H}): \alpha(T) = T\}$ [resp., $\bigcap_{n=1}^{\infty} \alpha^n[\mathcal{B}(\mathcal{H})]$] consists only of scalar multiples of the identity operator. The (Powers) index of $\alpha$ is defined to be the extended integer $n$ such that $\alpha[\mathcal{B}(\mathcal{H})]' \cap \mathcal{B}(\mathcal{H})$ is isomorphic to the factor of type $I_n$. Two unital endomorphisms $\alpha$ and $\beta$ are conjugate if there exists an automorphism $\gamma$ of $\mathcal{B}(\mathcal{H})$ such that $\alpha = \gamma \beta \gamma^{-1}$, and the authors say that $\alpha$ and $\beta$ are approximately conjugate if $\alpha$ is in the norm-closure of the set of all endomorphisms that are conjugate to $\beta$.

This paper, semi-expository in nature, gives a deep and penetrating analysis of the conjugacy problem for endomorphisms of $\mathcal{B}(\mathcal{H})$. The work of M. Laca is highlighted: Laca showed that if $n \in \{2, 3, \cdots, \infty\}$, the set of conjugacy classes of all endomorphisms of index $n$ can be parametrized by a natural Borel equivalence which is not countably separated (the same statement is valid for shifts and ergodic endomorphisms of index $n$). The authors give an independent proof of this theorem (for $n < \infty$) and describe an explicit set of labels for the conjugacy classes by exploiting the correspondence between endomorphisms of $\mathcal{B}(\mathcal{H})$ and representations of the Cuntz algebras which implement them. The Cuntz-algebra approach is also used to obtain interesting new results, among which are explicit examples of shifts with no invariant normal states, ergodic and clustering properties of endomorphisms of $\mathcal{B}(\mathcal{H})$, and a theorem which asserts that two such endomorphisms with the same index $n$, $2 \leq n < \infty$, are always approximately conjugate.

{For the entire collection see MR 96m:00020.}

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