The article continues previous work by the authors [J. Funct. Anal. 125 (1994), no. 1, 90–110; MR 95i:47067]. Here they consider affine systems in $\mathbb{R}^n$ constructed from quadruples $(R, B, L, K)$ where $R$ is an invertible integral matrix, $B$ and $L$ are finite subsets of $\mathbb{R}^n$ having the same cardinality $N$, and $K$ is a lattice in $\mathbb{R}^n$. The associated affine system is defined by $\sigma_b(x) = R^{-1}x + b$ for $b \in B$ and $x \in \mathbb{R}^n$. Under suitable assumptions, there exists a unique probability measure $\mu$ on $\mathbb{R}^n$ satisfying $\mu = |B|^{-1} \sum_b \mu \circ \sigma_b^{-1}$ and supported on a suitable “fractal” set $X$. It turns out that when the matrix $U = (e^{2\pi ib \cdot l})_{b,l}$ satisfies $U^*U = UU^* = N1_N$, one gets a pair of representations $(S_b)$ and $(T_l)$ of the Cuntz algebra $O_N$ acting on $L^2(\mu)$ such that $S_b^*T_l$ is a multiplication operator for all $b$ and $l$. This is a substitute for the classical harmonic analysis problem which consists in finding a subset $\Lambda$ of $\mathbb{R}^n$ such that the exponentials $(e^{i\lambda x})_{\lambda \in \Lambda}$ form an orthonormal basis for $L^2(\mu)$. The paper also contains examples of affine systems for $N \leq 4$ including graphic illustrations of the corresponding sets $X$. 

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