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Harmonic analysis and fractal limit-measures induced by representations of a certain $C^*$-algebra. (English. English summary)


Let $\Omega$ be a measurable subset of $\mathbb{R}^d$ with finite, positive Lebesgue measure, and let $\Lambda$ be a subset of $\mathbb{R}^d$ containing 0. For $\lambda \in \Lambda$, set $e_\lambda(x) = e^{i2\pi \lambda x}$ for $x \in \Omega$. Then $(\Omega, \Lambda)$ is called a spectral pair if $\{e_\lambda; \lambda \in \Lambda\}$ is an orthonormal basis of $L^2(\Omega)$. Given such a pair, the authors construct recursively a sequence of spectral pairs $(\Omega_j, \Lambda_j)_{j \geq 0}$ with $(\Omega_0, \Lambda_0) = (\Omega, \Lambda)$. Moreover, if $\mu_j$ is the measure on $\mathbb{R}^d$ defined by $\mu_j(B) = m(B \Omega_j)/m(\Omega_j)$ for every Borel set $B$ in $\mathbb{R}^d$, they get a fractal probability measure $\mu$ as a limit of $(\mu_j)$. Finally, two sets of isometries of $L^2(\mu)$ are defined, and they both provide representations of some Cuntz algebra. Conversely, such a measure may be reconstructed from suitable representations of Cuntz algebras.

See also the preceding review. 

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