2002c:46117 46L10 37C30 42A16 42A65 43A65 47B38

Jorgensen, Palle E. T.

Ruelle operators: functions which are harmonic with respect to a transfer operator. (English. English summary)

Mem. Amer. Math. Soc. **152** (2001), *no.* 720, *viii*+60 *pp.*

Let $N \geq 2$ be an integer. Denote by $\mathfrak{A}_N$ the (universal) C^*-algebra generated by two unitaries U and V satisfying $UVU^{-1} = V^N$. Motivated by wavelet analysis, the author studies a class of representations of $\mathfrak{A}_N$ that we describe below. The idea rests on a construction of scaling functions from low-pass filters due to S. G. Mallat [Trans. Amer. Math. Soc. 315 (1989), no. 1, 69–87; MR 90e:42046]. Before describing an account of the main result, we need some preliminary considerations.

First, there is a one-to-one correspondence between representations of $\mathfrak{A}_N$ (i.e. realizations of U and V as unitary operators U_π and V_π such that $U_\pi V_\pi U_\pi^{-1} = V_\pi^N$) and pairs (π, U_π), where π is a representation of $L^\infty(\mathbb{T})$ and U_π is a unitary operator such that

$$U_\pi \pi(f) U_\pi^{-1} = \pi(f(z^N)), \quad f \in L^\infty(\mathbb{T}).$$

In fact, $V_\pi = \pi(e_1)$, where $e_n(z) = z^n$, $n \in \mathbb{Z}$. Moreover, the representation is said to be normal if it admits a cyclic vector φ such that the spectral measure of V_π associated to φ is absolutely continuous with respect to Haar measure on $\mathbb{T}$.

Next, let $m_0 \in L^\infty(\mathbb{T})$ be a low-pass filter. The associated Ruelle transfer operator is defined on $L^1(\mathbb{T})$ by

$$(Rf)(z) = \frac{1}{N} \sum_{w^N = z} |m_0(w)|^2 f(w).$$

Then the main theorem of the article establishes a relationship between the operator R and normal representations of $\mathfrak{A}_N$ as follows: Suppose that we are given a normal representation π of $\mathfrak{A}_N$ with cyclic vector φ satisfying $U_\pi \varphi = \pi(m_0)\varphi$; define $h_\varphi \in L^1(\mathbb{T})$ by

$$h_\varphi(z) = \sum_{n \in \mathbb{Z}} z^n \langle \pi(e_n)\varphi | \varphi \rangle.$$

Then $Rh_\varphi = h_\varphi$. Conversely, if $h \in L^1(\mathbb{T})$, $h \geq 0$, is a solution to $Rh = h$, then there is a normal representation with cyclic vector φ such that $h = h_\varphi$. Moreover, one has uniqueness up to unitary equivalence.

Paul Jolissaint (CH-NCH)