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Jorgensen, Palle E. T. (1-IA)

A geometric approach to the cascade approximation operator for wavelets. (English. English summary)

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This paper is part of a program of investigating wavelets from an operator-theoretic and representation-theoretic viewpoint. Earlier work in that direction includes [O. Bratteli and P. E. T. Jorgensen, Integral Equations Operator Theory 28 (1997), no. 4, 382–443; MR 99k:46094b; P. E. T. Jorgensen, Mem. Amer. Math. Soc. 152 (2001), no. 720, viii+60 pp.]. The main focus of interest here is the convergence of the cascade approximation $M^n h$, where M is the cascade operator and h a suitable starting vector. In favorable cases the sequence converges to the scaling function for a multiresolution analysis.

We paraphrase from the author's abstract and introduction: Let $\mathcal{H}$ be a Hilbert space and π a representation of $L^\infty(\mathbb{T})$ on $\mathcal{H}$. Let R be a positive operator on $L^\infty(\mathbb{T})$ which fixes the constants and such that $\pi(f)M = M\pi(f(z^2))$ and $M^*\pi(f)M = \pi(R^*f)$. A complete orthogonal expansion of $\mathcal{H}$ is given which reduces π such that M acts as a shift on one part and the residual part is $\bigcap_n [M^n\mathcal{H}]$. This is a generalization of the classical Kolmogorov-Wold decomposition for isometries. Using this decomposition, results are obtained about convergence of the cascade algorithm in an abstract Hilbert space setting. In the last sections these results are used to give some applications for wavelet theory. *Richard Rochberg* (1-WASN)