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Operator commutation relations. Commutation relations for operators, semigroups, and resolvents with applications to mathematical physics and representations of Lie groups. (English)

Mathematics and Its Applications. Dordrecht-Boston-Lancaster: D. Reidel Publishing Company, a Member of Kluwer Academic Publishers Group. XVIII, 493 p. Dfl. 180.00; $ 69.00 (1984).

Dirac mentioned somewhere that "the noncommutation was really the dominant characteristic of Heisenberg's theory". As is well known, in the Heisenberg formalism of the quantum physics the "observables" are infinite matrices, whereas the Schrödinger formalism uses partial differential operators. These formalisms are the historical roots of two different types of suffcient conditions, which ensure that some formal commutation relation, dealt with in the present book, are mathematically correct. And the very existence of a certain number of formal commutation identities leads to a surprising unification (as well as to a significant range of applications), in spite of the use of two distinct collections of techniques. The typical commutation relation which is of interest in this work is the following

$$B\phi(A) = \sum\{(-1)^n/n!\phi^{(n)}(A)(adA)^n(B); 0 \leq n < \infty\},$$

where (A,B) is a pair of closable endomorphisms on a dense subspace of a Banach (or locally convex) space, ϕ (A) is the image of A under a certain operational calculus and $(adA)(B) = AB - BA$. In particular, if $\phi(z) = (\lambda - z)^{-1}$ or $\phi(z) = \exp(tz)$, one easily derives (formally) the other two commutation relations which are studied in this book. The authors give several suffcient conditions for commutation theory, in the context of matrix operators in sequence spaces (corresponding to the Heisenberg formalism) as well as for differential operators in function spaces (corresponding to the Schrödinger formalism). They use such results in the exponentiation theory, i.e. the construction of local and global representation of Lie groups from infinitesimal representations of their Lie algebras. In particular, a functional analytic proof of Palais' global integration theorem for Lie algebras is given.

The present work emphasizes the rôle played by the analysis of infinitesimal and global commutation relations for operators in different areas of mathematics (pure and applied). The authors have chosen the C^∞-vector approach, as opposed to analytic vectors, because of its suitability to general Banach and locally convex spaces. Many of the results have been obtained by the authors themselves. The book, which is clearly and systematically written, is addressed to all sorts of mathematicians, graduate students and researchers, especially to those interested in the applications of the operator theory to mathematical physics.

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Keywords : C^∞-vectors; closable operators; invariant domains; operational calculus; exponentiation theory; representation of Lie groups from infinitesimal representations of their Lie algebras; Palais' global integration theorem for Lie algebras

Classification :

*47B47 Derivations and linear operators defined by algebraic conditions
47-02 Research monographs (operator theory)
22E65 Infinite-dimensional Lie groups
47A60 Functional calculus of operators
17B15 Representations of Lie algebras, analytic theory

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