Densities of Frames.

Preliminary report.

The decomposition of signals as a linear combination of some fixed set of vectors \( V \) is at the heart of most techniques in signal processing. The most common choice of \( V \) is a basis for the space of signals; such a situation produces a unique decomposition for any signal. However, picking an overcomplete set of vectors for \( V \) can be advantageous: from the many ways to decompose a given signal, a choice can be made that is “efficient”. Frames are choices for \( V \) that are overcomplete but that otherwise resemble bases.

Often one tries to use the general structure of the signals to guide the choice of vectors \( \{f_i\} \) for the frame. This is the case in Weyl-Heisenberg and wavelet systems where one sets the \( f_i \) to be shifts in time and frequency of a single vector (Weyl-Heisenberg) or dilates and shifts in time of a single vector (wavelet). In the case of Weyl-Heisenberg systems, a beautiful density result gives a necessary condition for the system \( \{e^{2i\alpha_j}f(t - b_j)\}_j \) to be a frame in terms of the density of the points \((a_j, b_j)\) in the plane. In this talk we’ll give a general notion of density for frames (not necessarily Weyl-Heisenberg) and relate frame density to frame span. This work extends and unifies most of the known density results about frames. (Received September 30, 2002)