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Density of Weighted Wavelet Frames.

If $\psi \in L^2(\mathbb{R})$, $\Lambda$ is a discrete subset of the affine group $A = \mathbb{R}^+ \times \mathbb{R}$, and $w : \Lambda \to \mathbb{R}^+$ is a weight function, then the weighted wavelet system generated by $\psi$, $\Lambda$, and $w$ is $W(\psi, \Lambda, w) = \{ w(a, b)^{1/2} a^{-1/2} \psi(\frac{x}{a} - b) : (a, b) \in \Lambda \}$. In this paper we define lower and upper weighted densities $D^-(\Lambda)$ and $D^+(\Lambda)$ of $\Lambda$ with respect to the geometry of the affine group, and prove that there exist necessary conditions on a weighted wavelet system in order that it possesses frame bounds. We apply these results to oversampled affine systems (which include the classical affine and the quasi-affine systems as special cases), to co-affine wavelet systems, and to systems consisting only of dilations, obtaining some new results relating density to the frame properties of these systems. (Received September 23, 2002)