HW 9 [PG]. Page 254: Take a look at problems 1 to 8 and try to answer these questions by reading the chapter carefully. You do not need to turn in these problems.

Turn in the following exercises: Page 254-256: Chapter 10, Exercises 9, 11, 12, 14.

9. a. The null hypothesis is \( H_0 : \mu = 74.4 \text{ mmHg} \). b. The alternative hypothesis is \( H_A : \mu \neq 74.4 \text{ mmHg} \). c. The test statistic is

\[
z = \frac{84 - 74.4}{9.1/\sqrt{10}} = 3.34.
\]

The p-value \( p < 0.002 \). d. Since \( p < 0.002 \), we reject the null hypothesis. In fact, the diastolic blood pressure for the female diabetics tend to be higher. e. Since \( p < 0.01 \), the conclusion would be the same.

11. a. We use the \( t \)-distribution with \( df = 58 - 1 = 57 \). A two-sided confidence interval is:

\[
(25.0 - 2.000 \times \frac{2.7}{\sqrt{58}}, 25.0 + 2.000 \times \frac{2.7}{\sqrt{58}}) = (24.3, 25.7).
\]

b. \( H_0 : \mu = 24.0, \quad H_A : \mu \neq 24.0. \)

The test statistic is

\[
t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{25.0 - 24.0}{2.7/\sqrt{58}} = 2.82.
\]

The p-value: \( 0.001 < p < 0.01 \). Therefore, we reject \( H_0 \).

c. The mean baseline body mass index for the population of men who later developed diabetes mellitus is higher than 24.0 kg/m\(^2\).

d. Since the value 24.0 is not contained in the 95\% confidence interval for \( \mu \), we would expect that the null hypothesis would be rejected.

12. a. \( H_0 : \mu = 136 \text{ mmHg}, \quad H_A : \mu \neq 136 \text{ mmHg}. \)

The test statistic is

\[
t = \frac{\bar{x} - \mu_0}{s_s/\sqrt{n}} = \frac{143 - 136}{24.4/\sqrt{86}} = 2.66.
\]

For a \( t \)-distribution with 85 df, \( 0.001 < p < 0.01 \). We reject \( H_0 \) at the 0.10 level of significance.

b. \( H_0 : \mu = 84 \text{ mmHg}, \quad H_A : \mu \neq 84 \text{ mmHg}. \)
The test statistic is

\[ t = \frac{\bar{x} - \mu_0}{s_0/\sqrt{n}} \]

\[ = \frac{87 - 84}{16.0/\sqrt{86}} \]

\[ = 1.74. \]

For a \( t \)-distribution with 85 df, 0.05 < \( p < 0.10 \). We reject \( H_0 \) at the 0.10 level of significance.

c. The workers who have experienced a major coronary event have a higher level of mean systolic blood pressure and a higher level of diastolic blood pressure than workers who do not.

14. a. The type I error rate = \( \alpha = 0.05 \). b. First, the critical value \( \bar{x} \) satisfies:

\[ \frac{\bar{x} - 244}{41/\sqrt{n}} = -1.645. \]

Therefore,

\[ \bar{x} = 230.5. \]

That is, the null hypothesis will be rejected if the sample has a mean no greater than 230.5 mg/100ml. Under the alternative hypothesis value \( \mu = 219 \), we have

\[ z = \frac{230.5 - 219}{41/\sqrt{25}} = 1.40. \]

The area to the right of 1.40 under the standard normal curve is 0.081. Therefore, the type II error rate \( \beta = 0.081 \).

c. The power

\[ \text{power} = 1 - \beta = 1 - 0.081 = 0.919. \]

d. The power can be increased by increasing the sample size, by increasing the type I error rate \( \alpha \), or by considering a hypothesized mean that is larger than 244 that is further away from 219.

e. For \( \alpha = 0.05 \) and \( \beta = 0.05 \), the required sample size

\[ n = \left[ \frac{(z_{\alpha} + z_{\beta})\sigma}{\mu_1 - \mu_0} \right]^2 = \left[ \frac{(1.645 + 1.645)41}{244 - 219} \right]^2 = 29.1. \]

Thus sample size of 30 would be needed.

f. If the type I error rate \( \beta = 0.10 \), then

\[ n = \left[ \frac{(z_{\alpha} + z_{\beta})\sigma}{\mu_1 - \mu_0} \right]^2 = \left[ \frac{(1.645 + 1.280)41}{244 - 219} \right]^2 = 23.0. \]

Thus sample size of 23 would be needed.