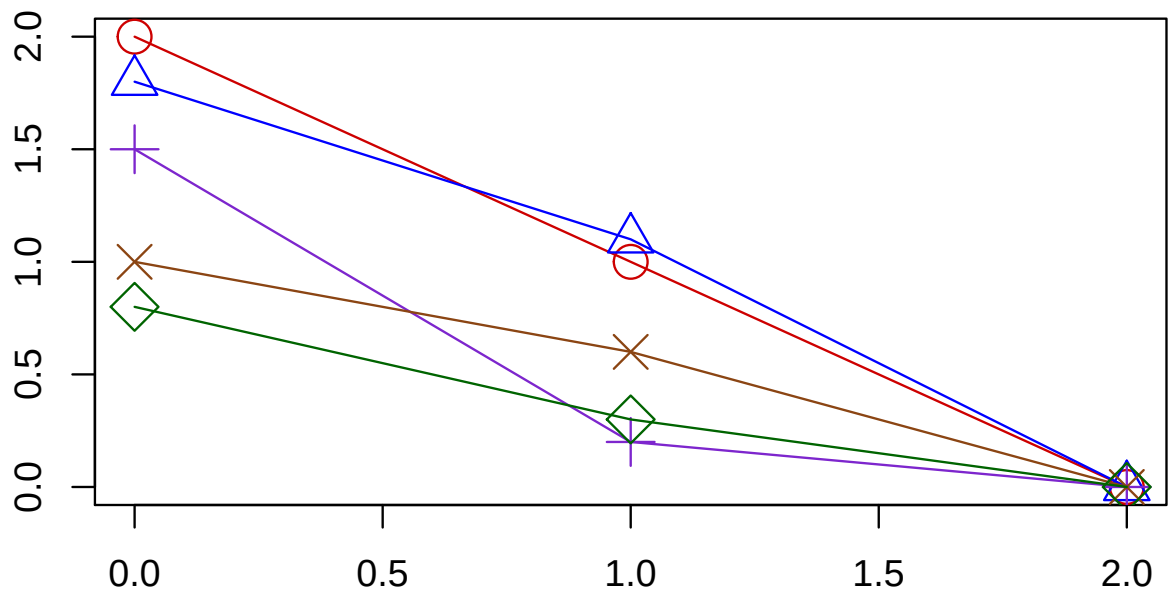
Suppose we have the following feature vectors:

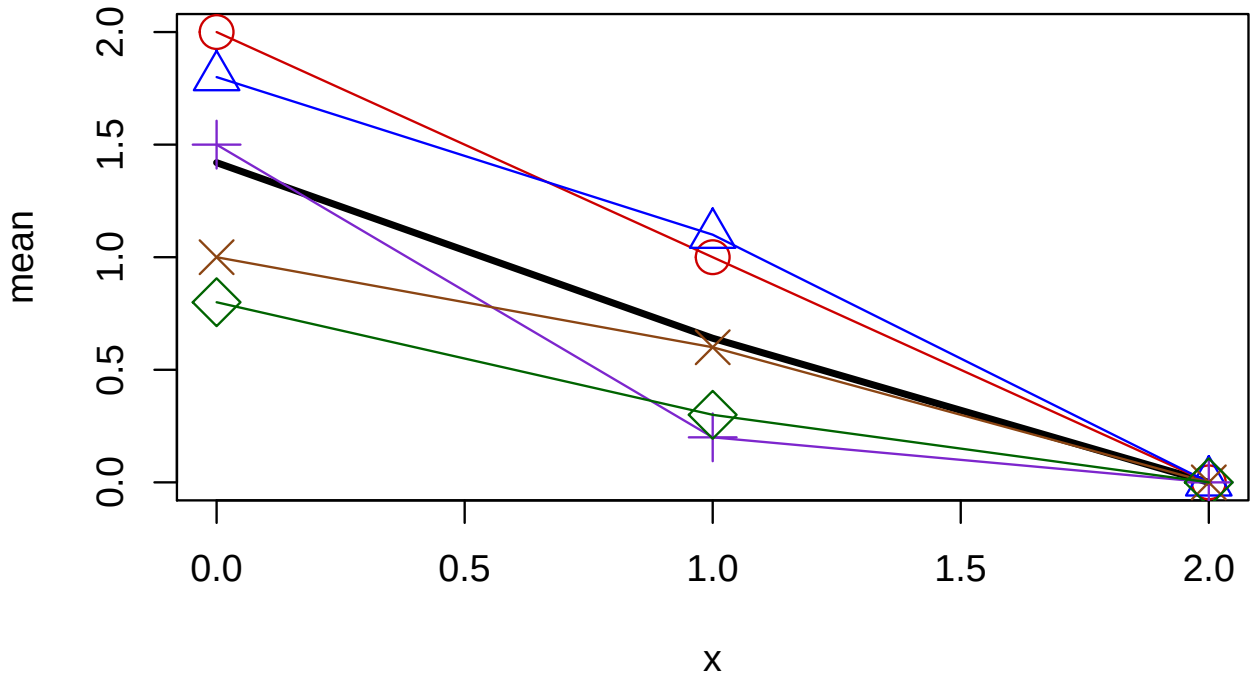
	f(0)	f(1)	f(2)
red	2.0	1.0	0
blue	1.8	1.1	0
purple	1.5	0.2	0
chocolate	1.0	0.6	0
green	0.8	0.3	0

We can plot these feature vectors (regardless of their length) in $\mathbb{R}^2$



Let's take the mean of our feature vectors and plot it's graph in black:

```
> Y<- rbind(y1, y2, y3, y4, y5)
> mean <- colMeans(Y)
> XY <- cbind(x, mean)
> t(XY)
     [,1] [,2] [,3]
x    0.00 1.00    2
mean 1.42 0.64    0
```



To see how these functions (feature vectors) vary about the mean, we will subtract the mean vector from each feacture vector:

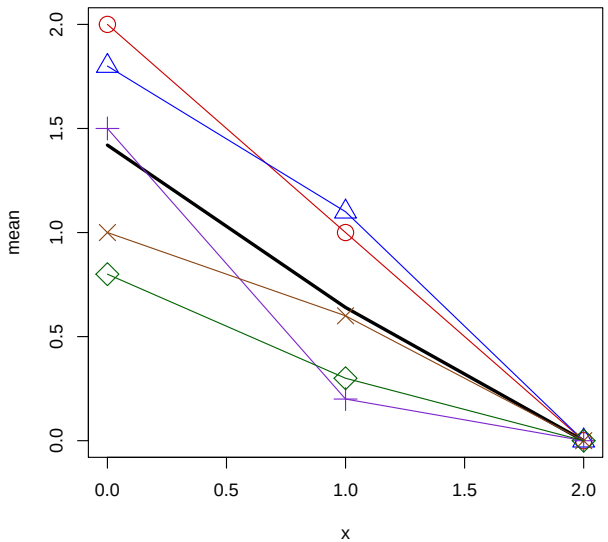


Figure : Raw Data + Mean

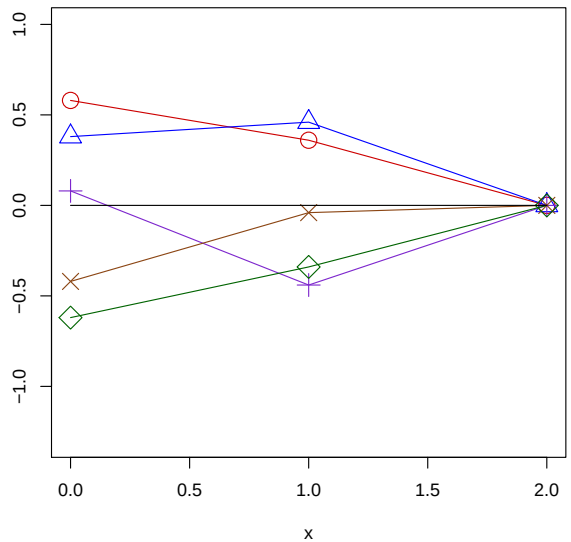
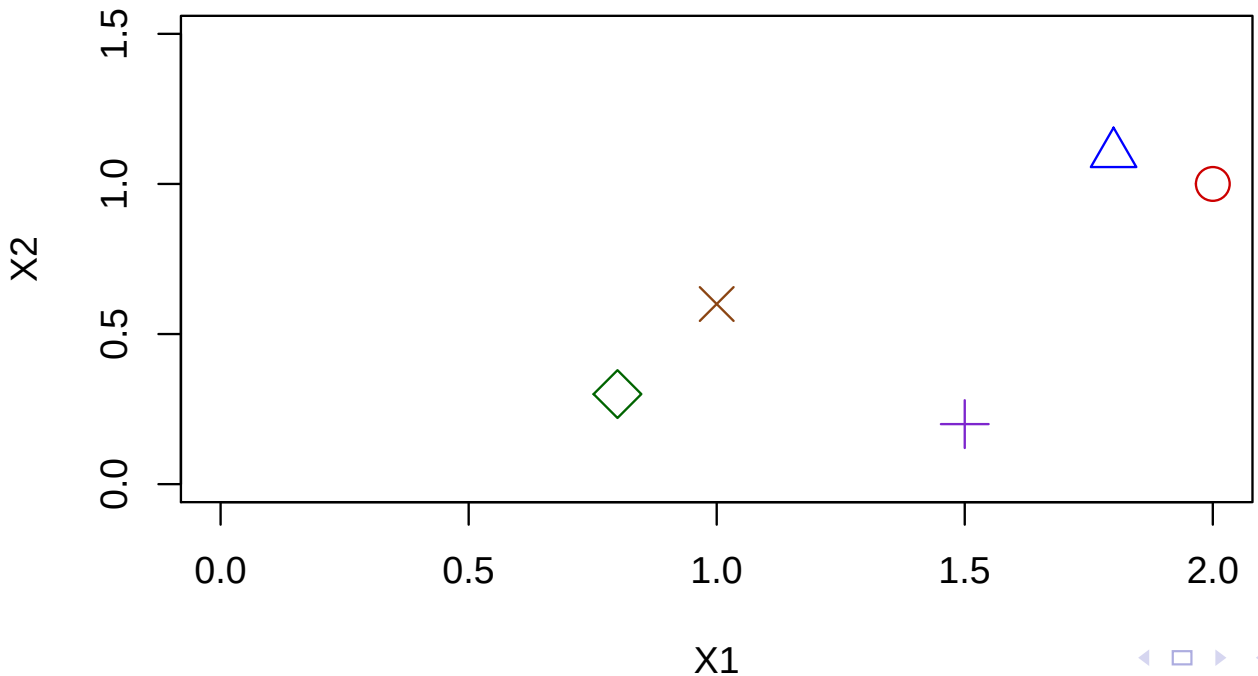


Figure : Mean Residuals

We can also plot our feature vectors in $\mathbb{R}^n$ where n is the number of features

	f(0)	f(1)	f(2)
red	2.0	1.0	0
blue	1.8	1.1	0
purple	1.5	0.2	0
chocolate	1.0	0.6	0
green	0.8	0.3	0



Let's translate these points by subracting off the mean:

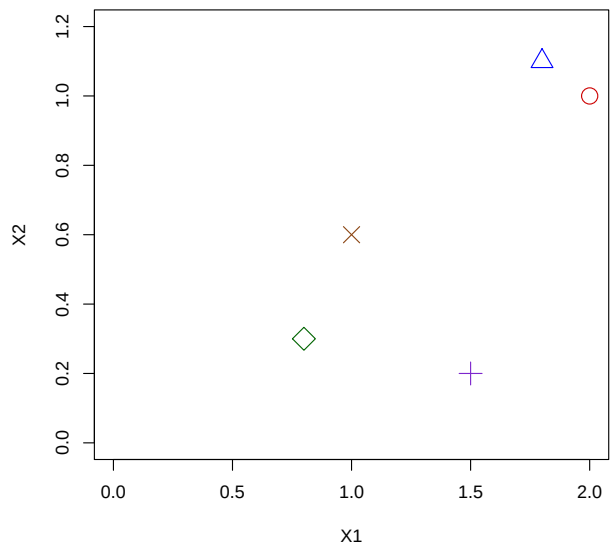


Figure : Raw Data + Mean

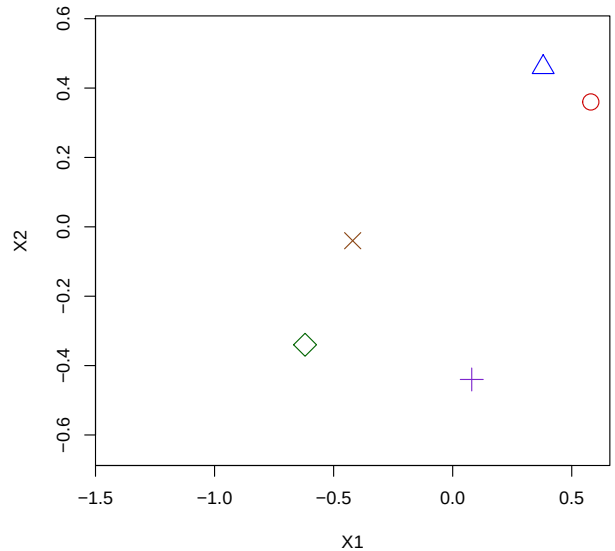


Figure : Centered about the origin

Covariance:

$$Cov(X, Y) = \sum_{i=0}^{N} \frac{(x_i - \overline{x})(y_i - \overline{y})}{N}$$

```
    X1  X2
y1 2.0 1.0
y2 1.8 1.1
y3 1.5 0.2
y4 1.0 0.6
y5 0.8 0.3
```

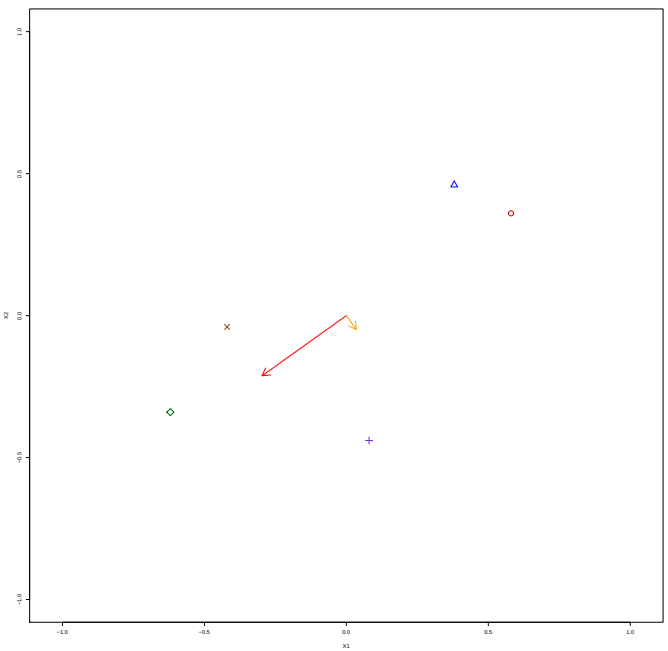
covariance matrix:

```
       X1    X2
X1 0.262 0.144
X2 0.144 0.163
```

We can now apply PCA to our data:

Covariance matrix:

```
       X1    X2
X1 0.262 0.144
X2 0.144 0.163
```



Eigenvalues and Eigenvectors

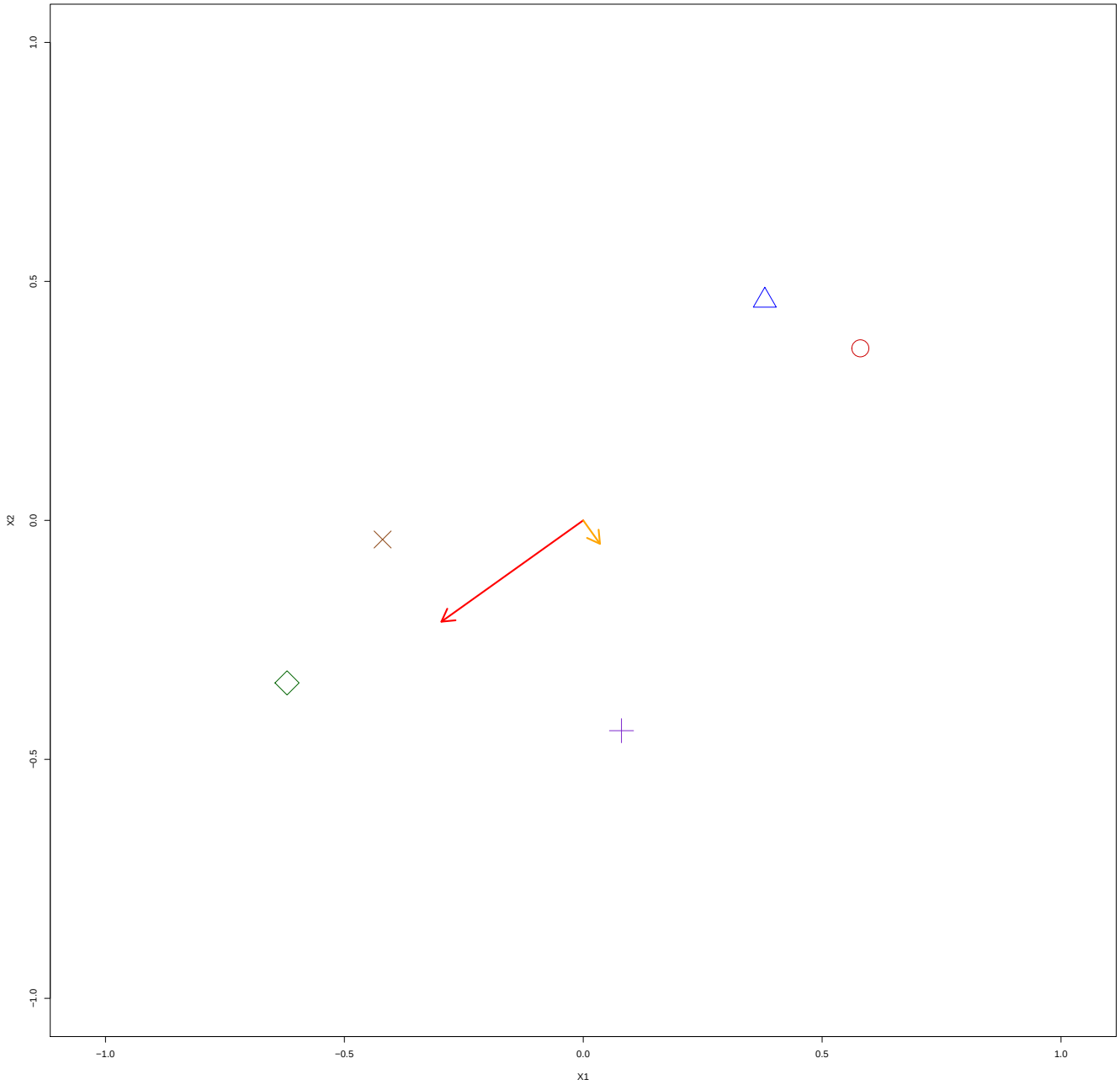
```
$values
[1] 0.36477032 0.06022968

$vectors
            [,1]       [,2]
[1,] -0.8139655  0.5809132
[2,] -0.5809132 -0.8139655
```

```
> E$values[1]/(E$values[1]+E$value
```

```
[1] 0.8582831
```

```
> sqrt(E$values)
```

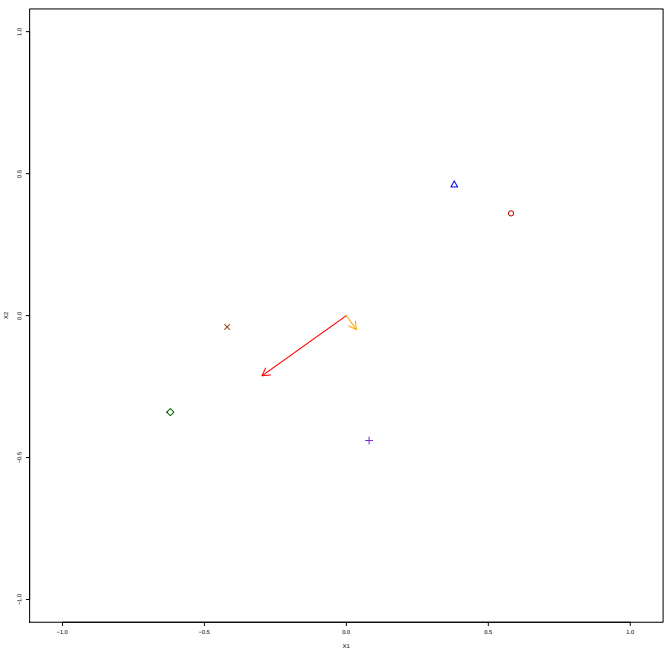
```
[1] 0.6039622 0.2454174
```


1.0
0.5
0.0
−0.5
−1.0
X2
−1.0
−0.5
0.0
0.5
1.0
X1

We can now apply PCA to our data:

Covariance matrix:

```
       X1    X2
X1 0.262 0.144
X2 0.144 0.163
```



Eigenvalues and Eigenvectors

```
$values
[1] 0.36477032 0.06022968

$vectors
            [,1]       [,2]
[1,] -0.8139655  0.5809132
[2,] -0.5809132 -0.8139655
```

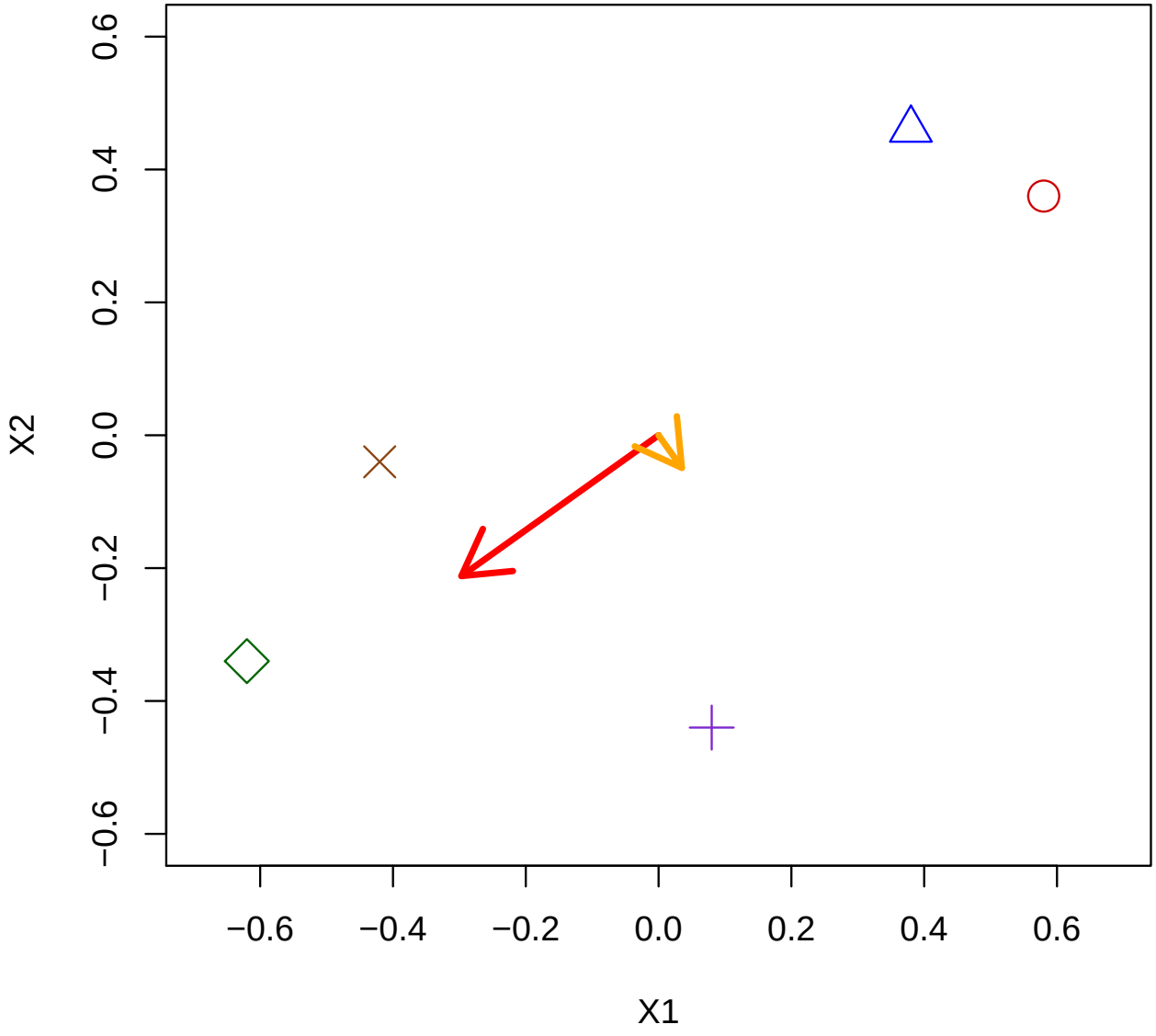
```
> E$values[1]/(E$values[1]+E$value
```

```
[1] 0.8582831
```

```
> sqrt(E$values)
```

```
[1] 0.6039622 0.2454174
```

```
> Y3<- as.matrix(t(t(Y2) - c(mean[1], mean[2])))
> prin_comp <- prcomp(Y3, scale=FALSE)
> prin_comp

Standard deviations:
[1] 0.6039622 0.2454174


Rotation:
          PC1        PC2
X1 -0.8139655  0.5809132
X2 -0.5809132 -0.8139655

> summary(prin_comp)

Importance of components:
                          PC1    PC2
Standard deviation     0.6040 0.2454
Proportion of Variance 0.8583 0.1417
Cumulative Proportion  0.8583 1.0000
```


X2
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
X1

```
[1] 0.6039622 0.2454174
```

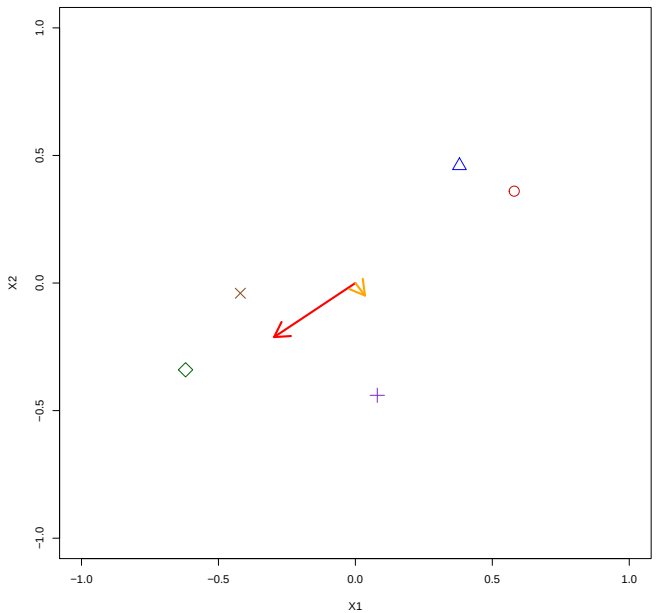
```
           PC1         PC2
X1 -0.8139655  0.5809132
X2 -0.5809132 -0.8139655
```

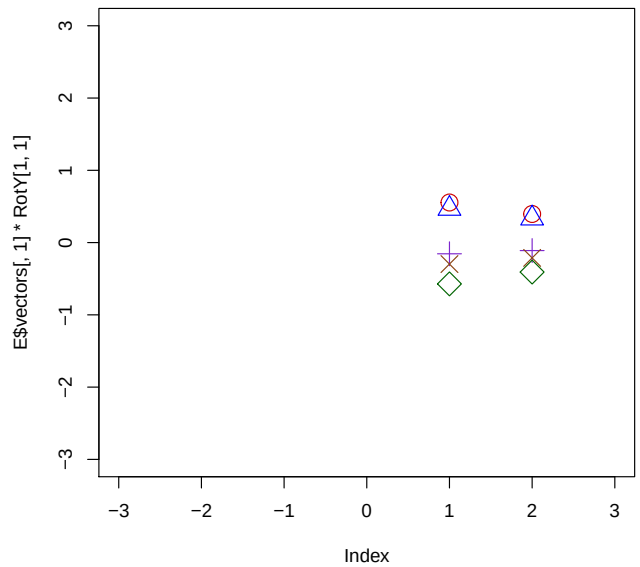
```
           PC1          PC2
y1 -0.6812288  0.04390205
y2 -0.5765270 -0.15367714
y3  0.1904846  0.40461789
y4  0.3651020 -0.21142491
y5  0.7021691 -0.08341789
```

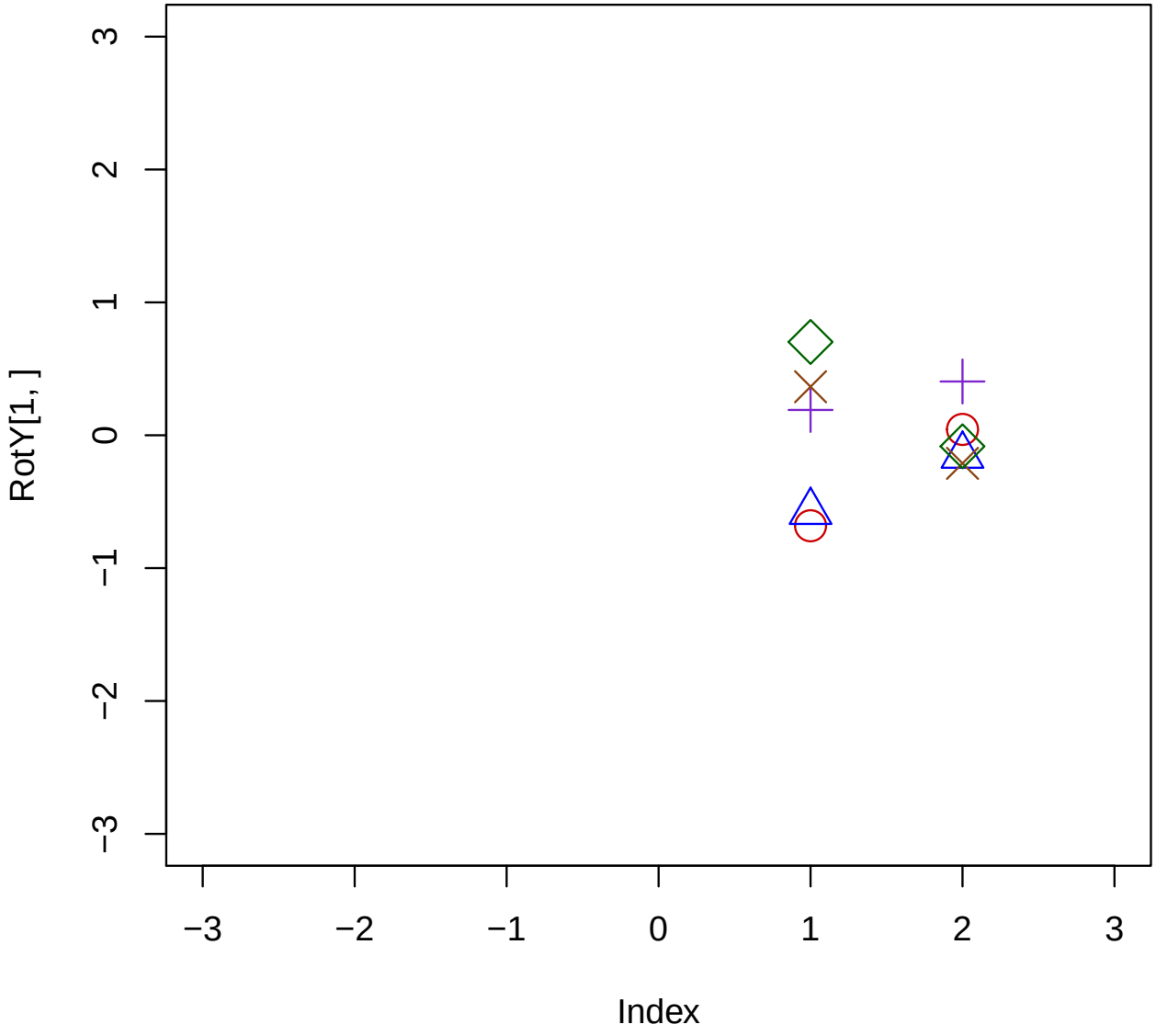
```
      X1    X2
y1  0.58  0.36
y2  0.38  0.46
y3  0.08 -0.44
y4 -0.42 -0.04
y5 -0.62 -0.34
```

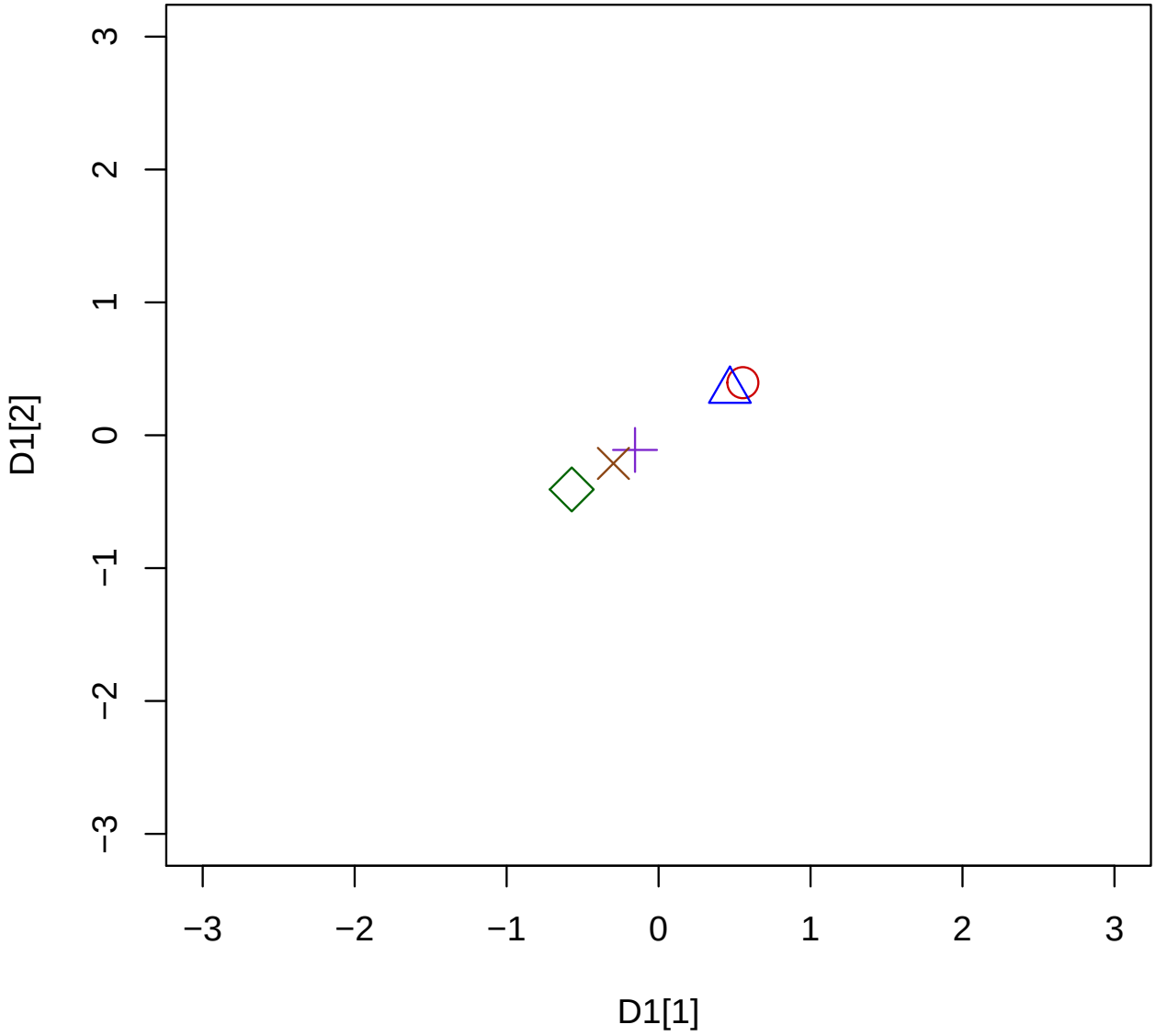
Covariance matrix:

```
       X1    X2
X1 0.262 0.144
X2 0.144 0.163
```





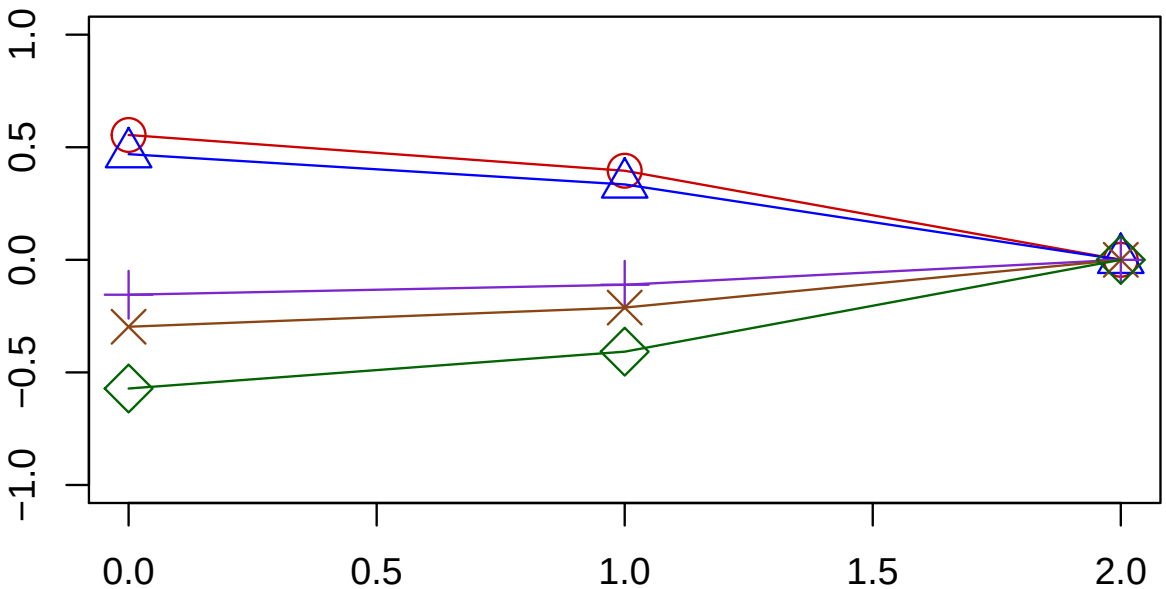

RotY[1,]
Index
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3


D1[2]
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3
D1[1]

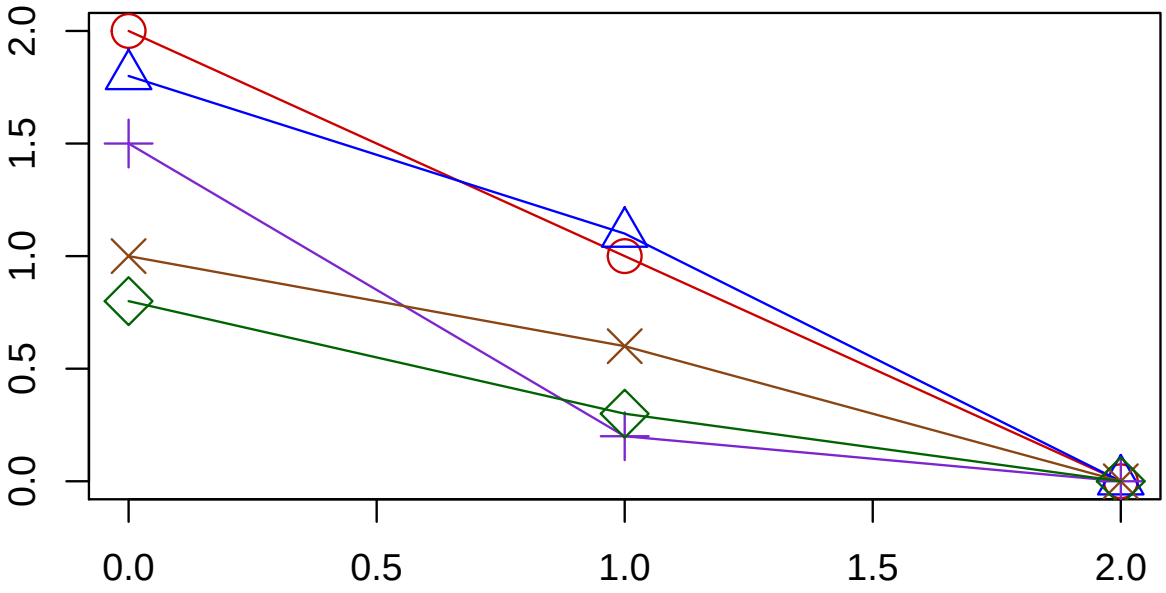
Suppose we have the following feature vectors:

```
                                                      colorY
[1,] "0.55449672074308"   "0.395734755466423"  "0" "red"
[2,] "0.469273073857219"  "0.334912107110384"  "0" "blue"
[3,] "-0.155047861106629" "-0.110654986955375" "0" "purple"
[4,] "-0.297180483373417" "-0.212092590483801" "0" "chocola
[5,] "-0.571541450120252" "-0.407899285137631" "0" "green"
```

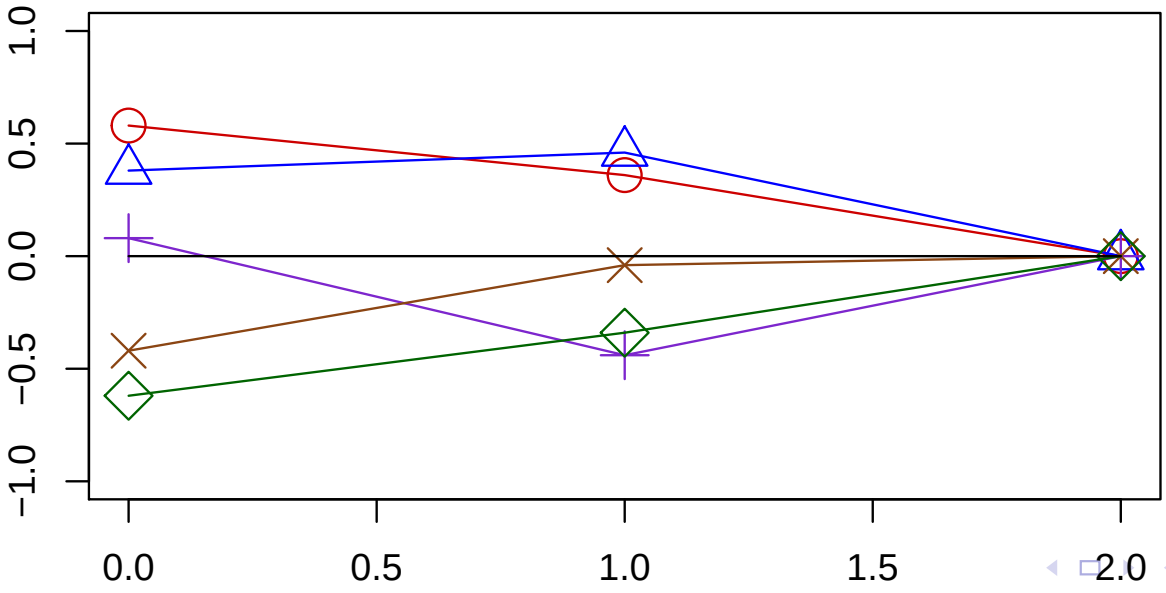
We can plot these feature vectors (regardless of their length) in $\mathbb{R}^2$



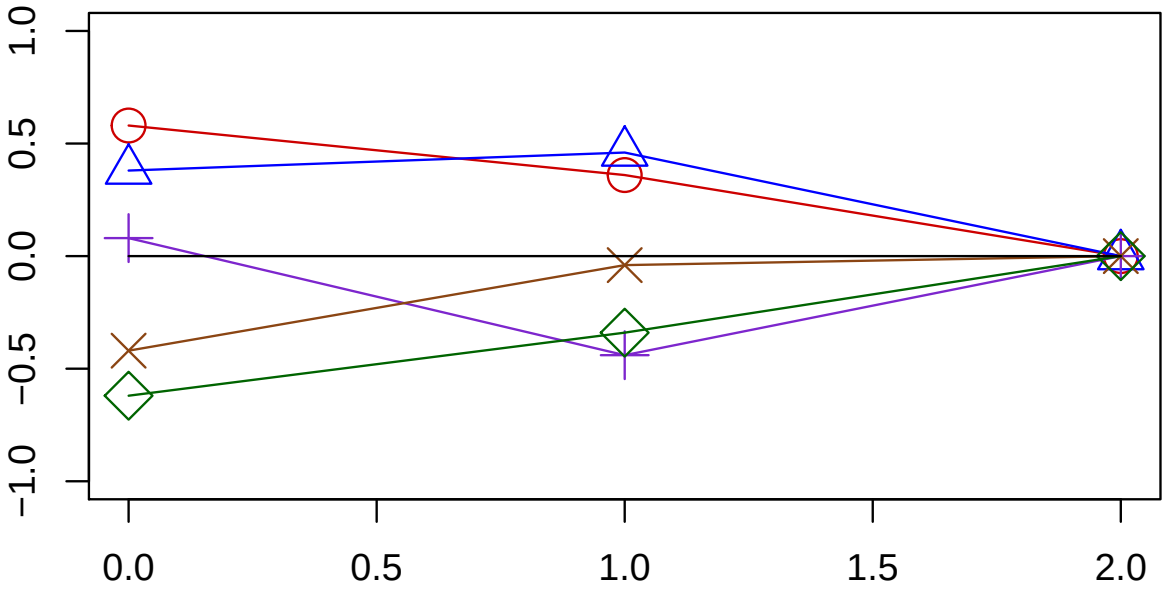
Raw Data



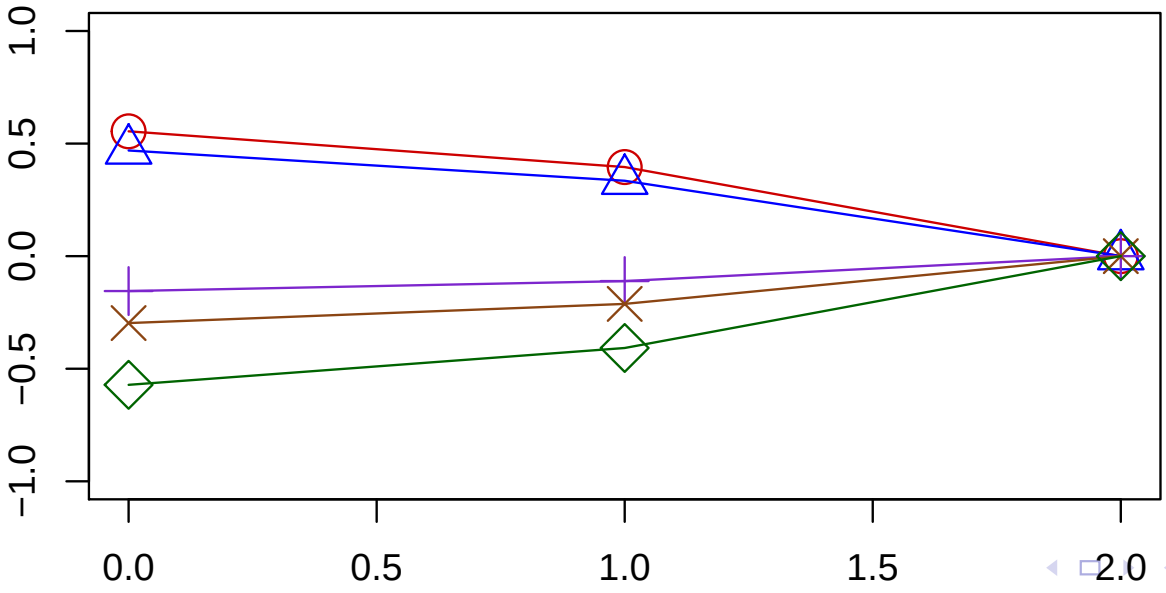
Mean Residual

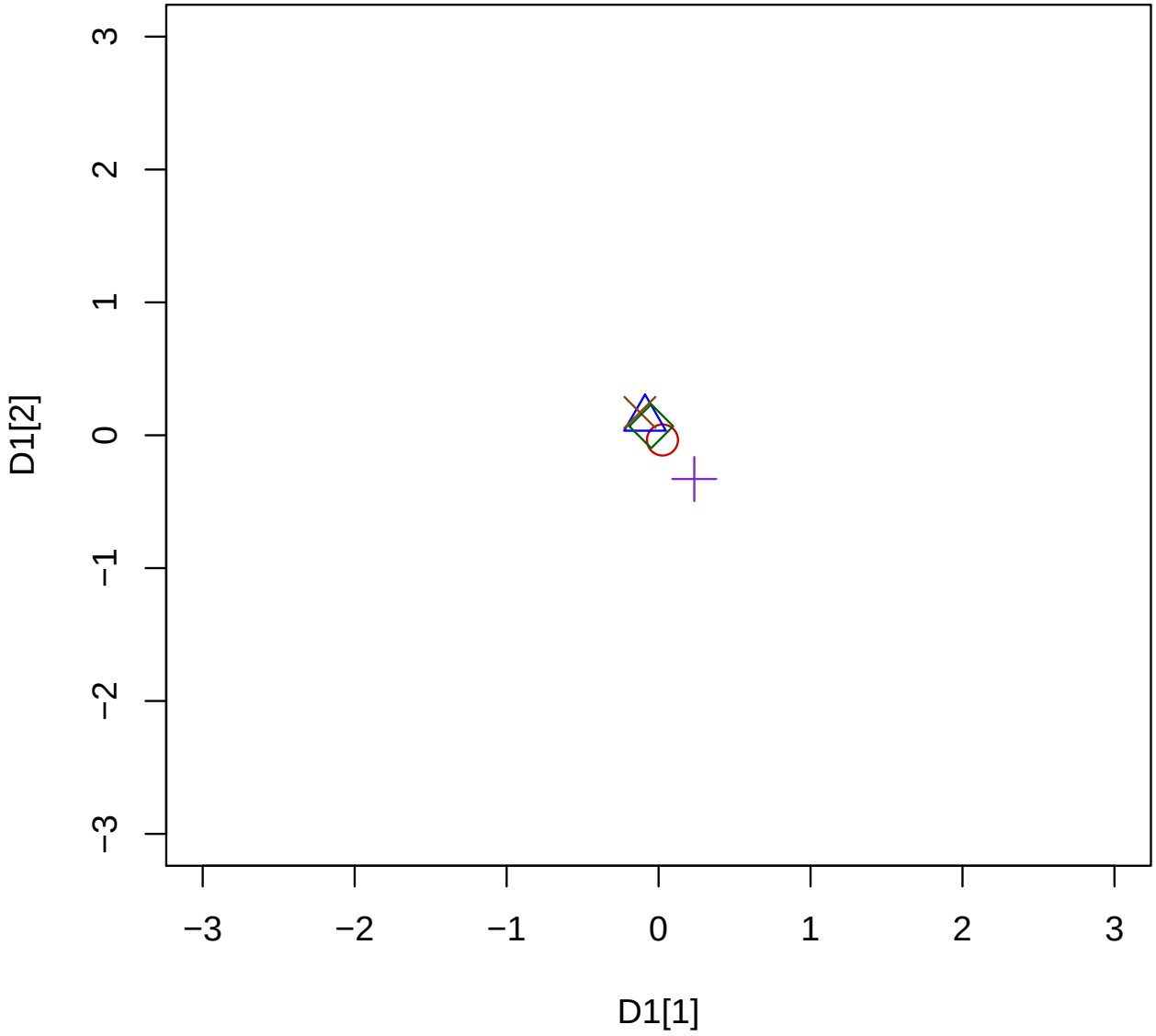


Mean Residual



PC1

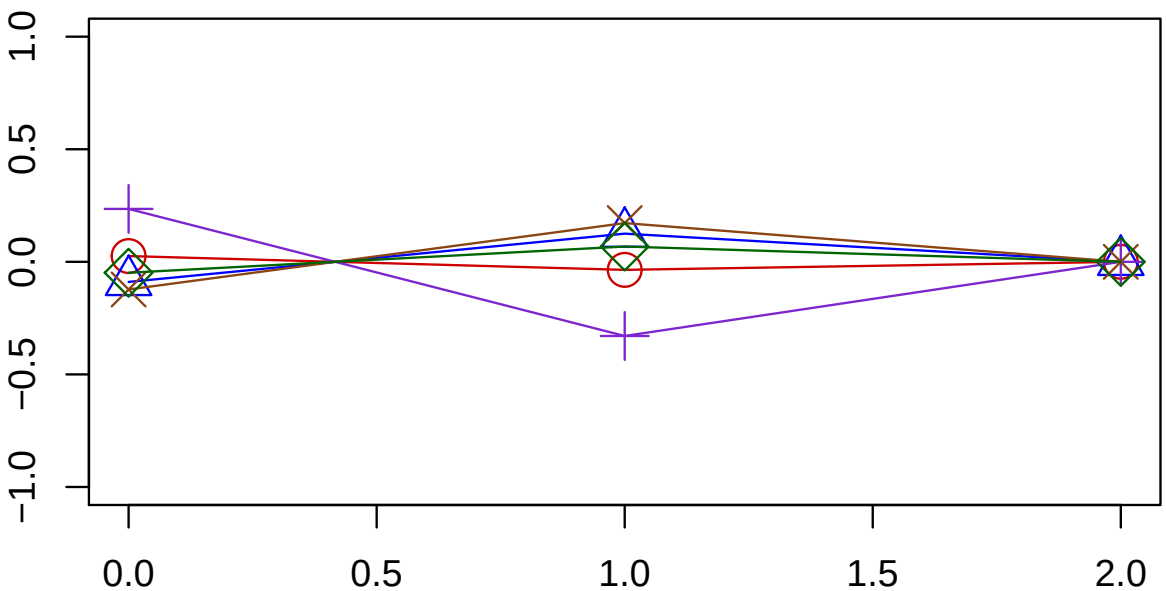



D1[2]
3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3
D1[1]

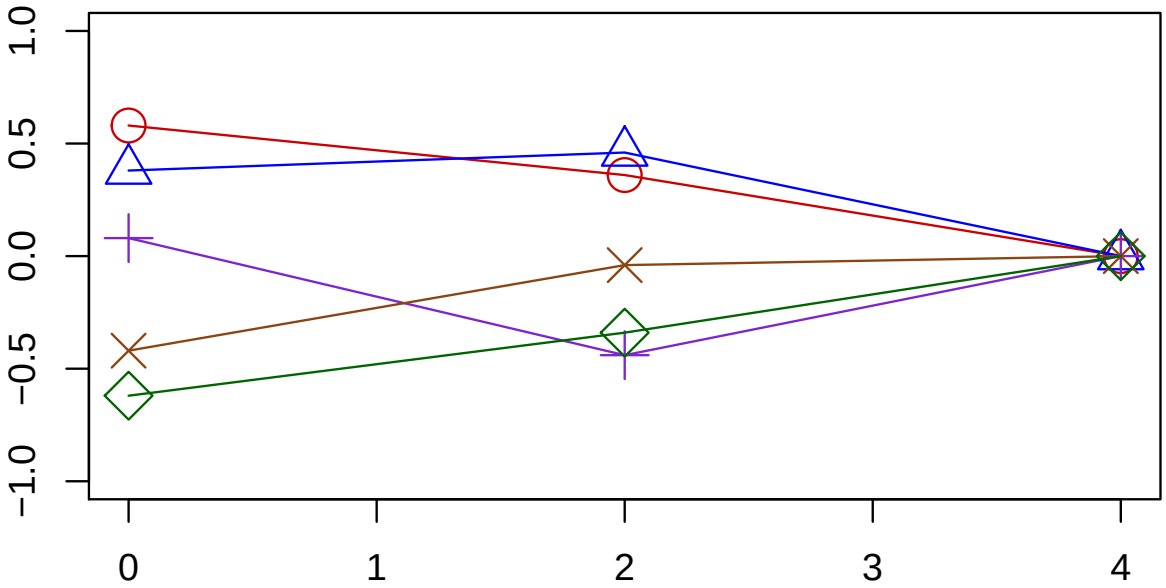
Suppose we have the following feature vectors:

```
                                                                  colorY
[1,] "0.0255032792569201"  "-0.0357347554664227" "0" "red"
[2,] "-0.0892730738572185" "0.12508792889616"   "0" "blue"
[3,] "0.235047861106629"   "-0.329345013044625" "0" "purpl
[4,] "-0.122819516626583"  "0.172092590483801"  "0" "choco
[5,] "-0.0484585498797477" "0.0678992851376303" "0" "green
```

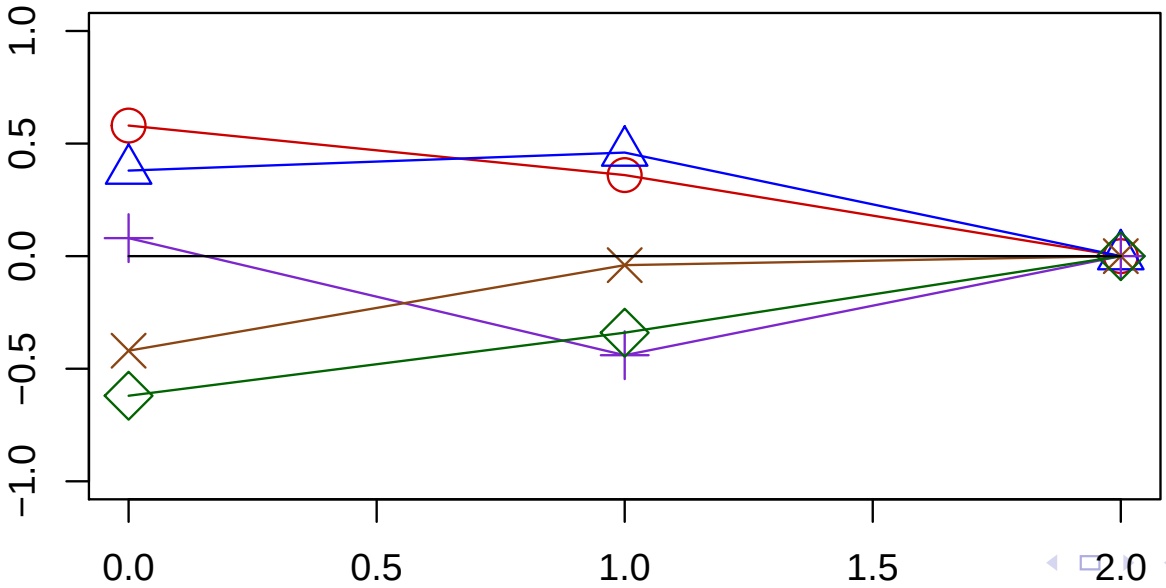
We can plot these feature vectors (regardless of their length) in $\mathbb{R}^2$

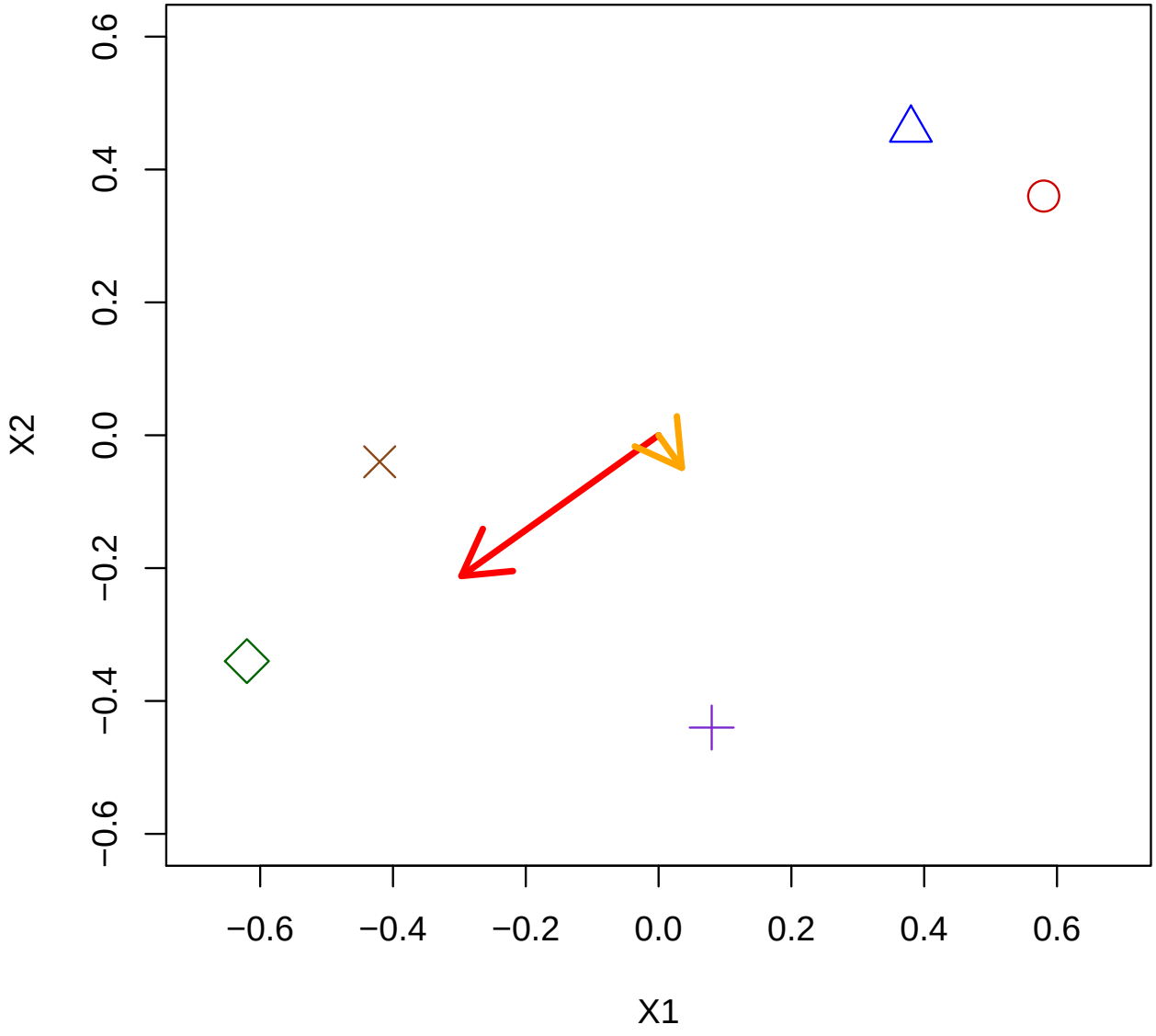


PC1 + PC2

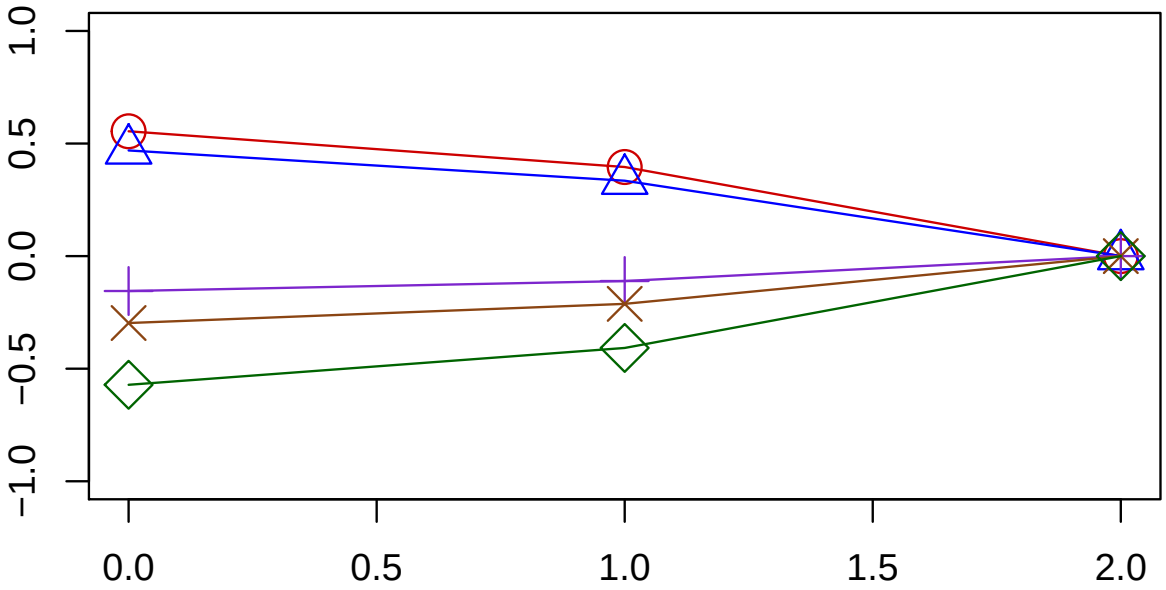


Mean Residual




X2
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.6
−0.4
−0.2
0.0
0.2
0.4
0.6
X1

PC1



PC2