Sheaf

A sheaf is a functor from from the category of simplicial complexes

Example: $\sigma \quad \rightarrow \quad F(\sigma)$, a vector space

$\rho \subset \sigma \quad \rightarrow \quad L : F(\rho) \rightarrow F(\sigma)$, a linear map

$F(\sigma)$ is called a **stalk**.

$F(\rho \subset \sigma)$ is called a **restriction map**

Let X be a simplicial complex.

For every σ, choose $s_\sigma \in F(\sigma)$. This assignment $(s_\sigma)_{\sigma \in X}$ of an element of $F(\sigma)$ to every simplex is called a **global section** if these choices are compatible with the restriction maps.

$F_X(X) =$ set of all global sections.

Example: The constant sheaf

Let X be a simplicial complex

The constant sheaf G_X:

$$\sigma \quad \to \quad G$$

$$\rho \subset \sigma \quad \to \quad id : G \to G$$

Suppose $(s_\sigma)_{\sigma \in X}$ is a global section.

Then if $s_\tau = g \in G$, then $s_\sigma = g$ for all $\sigma \in X$.

The the (group/vector space/...) of global sections in isomorphic to G.

Example: The skyscraper sheaf

The skyscraper sheaf G_τ

$$\sigma \quad \to \quad \begin{cases} G & \text{if } \sigma = \tau \\ 0 & \text{otherwise.} \end{cases}$$

$$\sigma \subset \sigma \quad \to \quad \textit{identity map}$$

$$\rho \subset \sigma \quad \to \quad \textit{zero map} \text{ if } \rho \neq \sigma.$$

Suppose $(s_\sigma)_{\sigma \in X}$ is a global section.

Then $s_\sigma = 0$ for all $\sigma \neq \tau$, since $s_\sigma \in F(\sigma) = 0$

If $dim \tau > 0$, then $\exists \rho \subset \tau$ and $F(\rho \subset \tau) = 0$. Thus $s_\tau = 0$.

Thus the (group/vector space/...) of global sections $\simeq \{0\}$.

If $dim \tau > 0$, let $s_\tau = g$. Thus in this case, the (group/vector space/...) of global sections $\simeq G$.

Direct sum

If F and G are sheaves, then $F \bigoplus G$ is a sheaf.

$$\sigma \quad \to \quad F(\sigma) \bigoplus G(\sigma)$$

$$\rho \subset \sigma \quad \to \quad F(\rho \subset \sigma) \bigoplus G(\rho \subset \sigma)$$

Example: $\bigoplus_{\tau \in X} G_\tau$

$$\sigma \quad \to \quad \bigoplus_{\tau \in X} G_\tau = G$$

$$\rho \subset \sigma \quad \to \quad \bigoplus_{\tau \in X} G_\tau(\rho \subset \sigma) = \begin{cases} \textit{identity map} & \rho = \sigma \\ \textit{zero map} & \rho \subset \sigma \end{cases}$$

Thus the (group/vector space/...) of global sections $\simeq \bigoplus_{v \in X} G$ since we can assign any element of G to a vertex v.

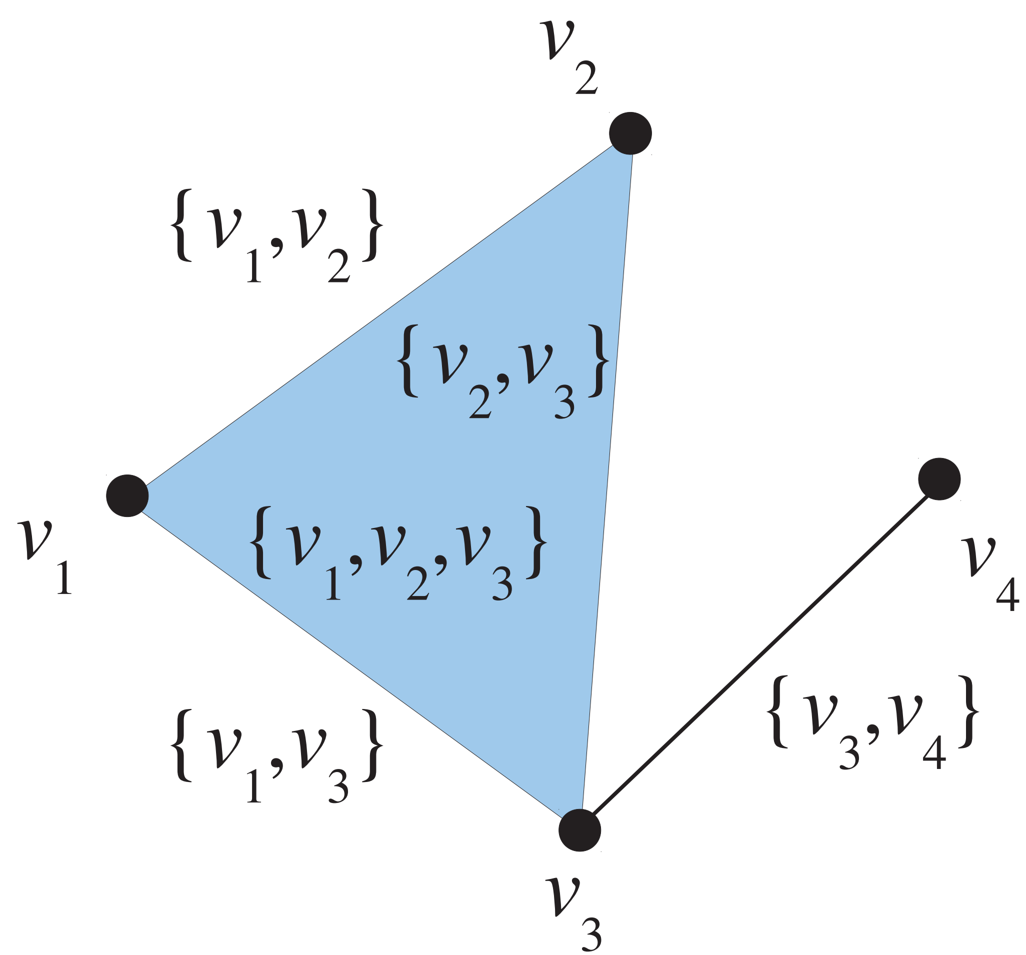
Now, abstractly...

A *sheaf* of ________________ on a _________________

(data type) (topological space)

Simplicial complexes

- … higher dimensional *simplices* (tuples of vertices)



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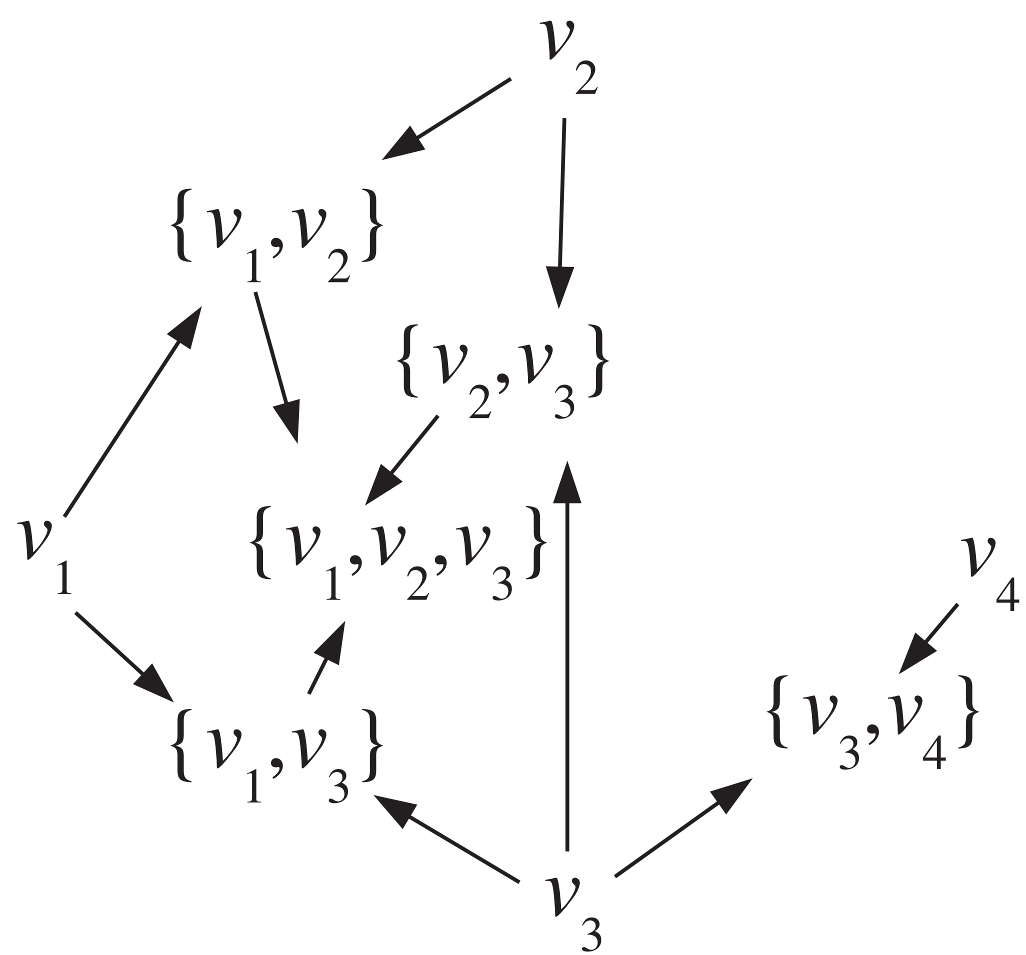
Simplicial complexes

- The *attachment diagram* shows how simplices fit together



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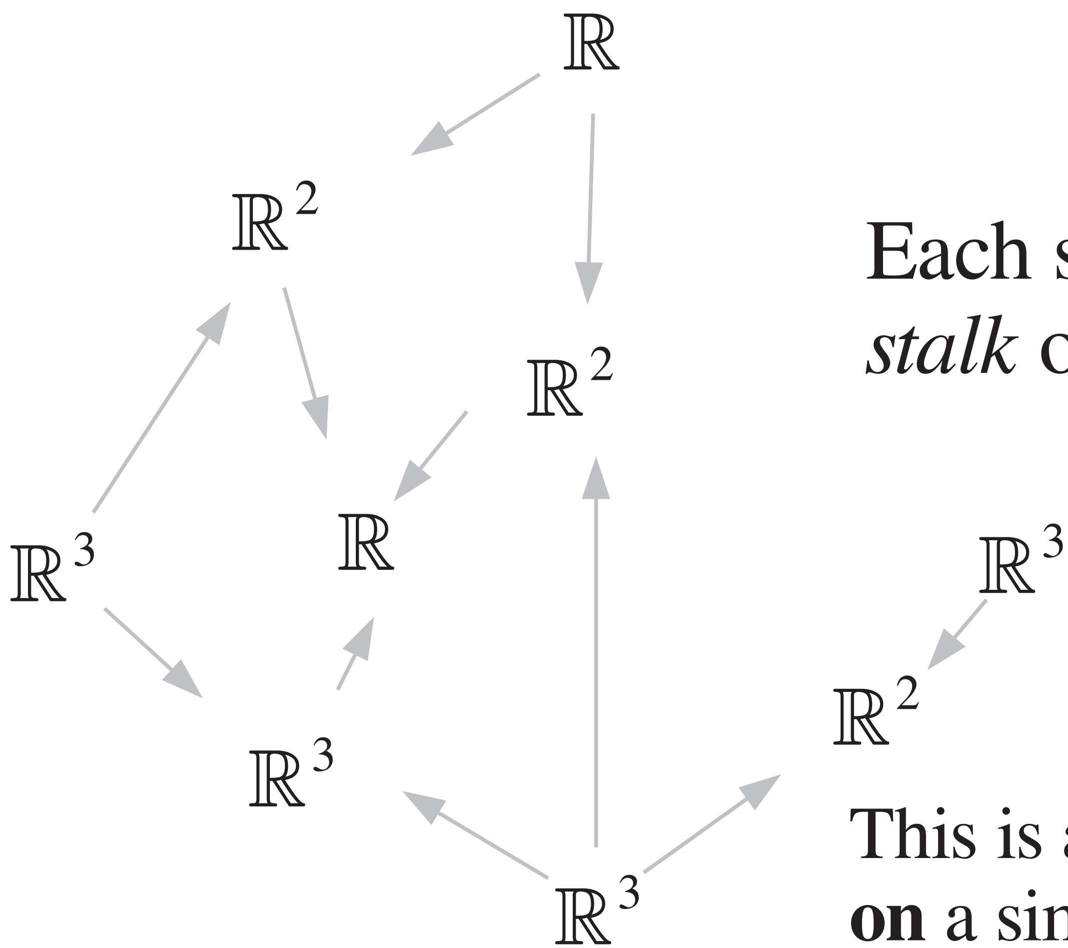
A *sheaf* is ...

- A set assigned to each simplex and ...

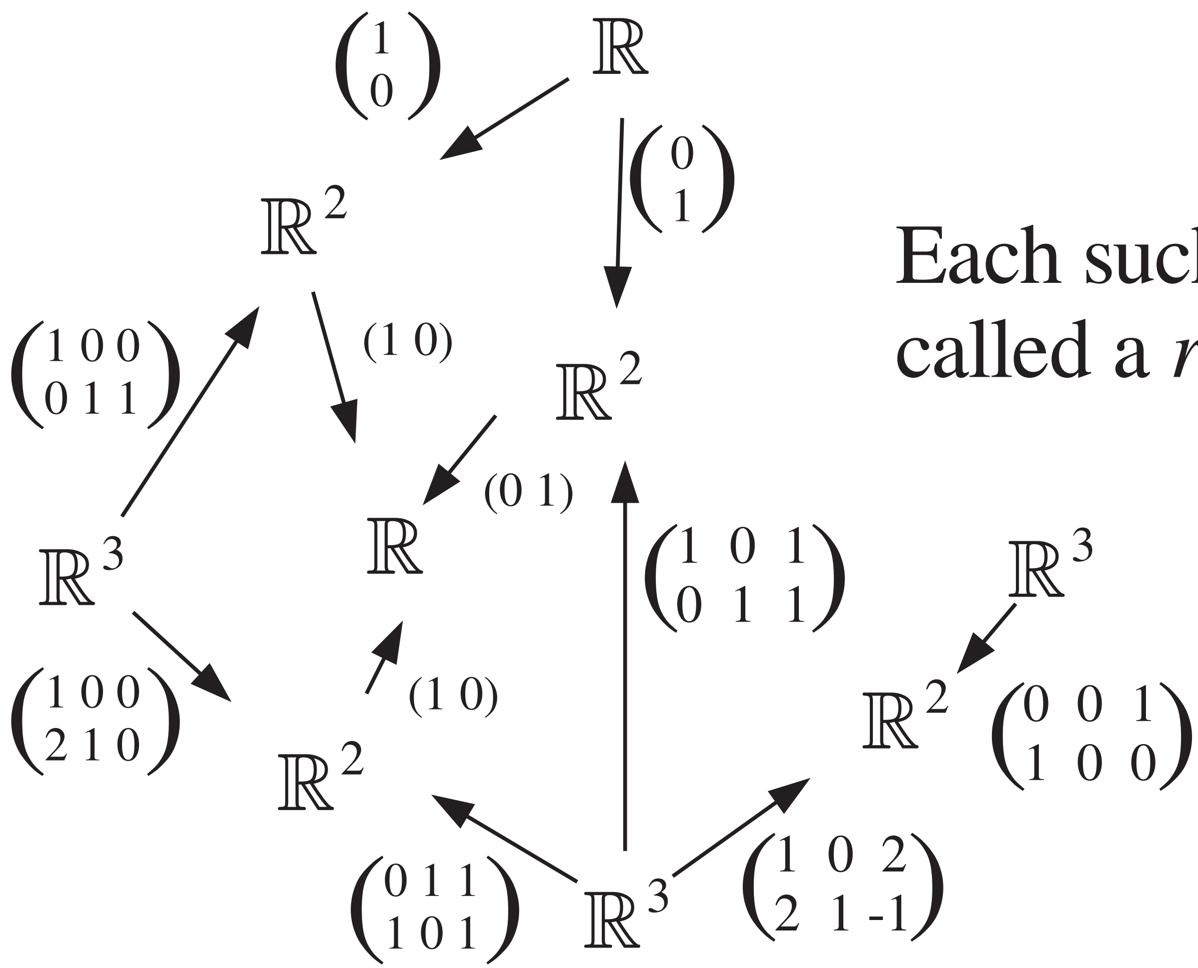


Each such set is called the *stalk* over its simplex

This is a sheaf **of** vector spaces **on** a simplicial complex

A *sheaf* is ...

- … a function assigned to each simplex inclusion



Each such function is called a *restriction*

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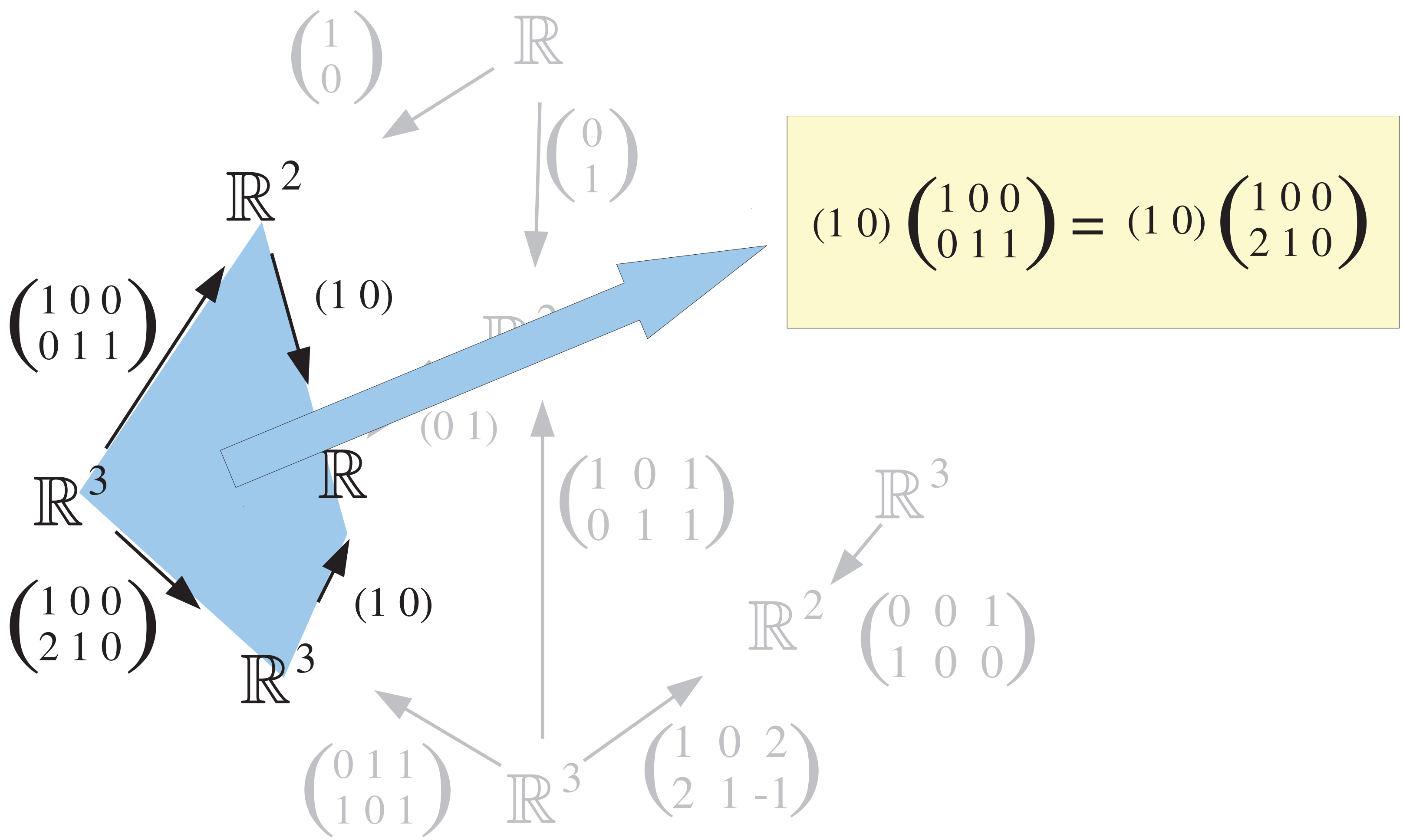
A *sheaf* is ...

- … so the diagram commutes.



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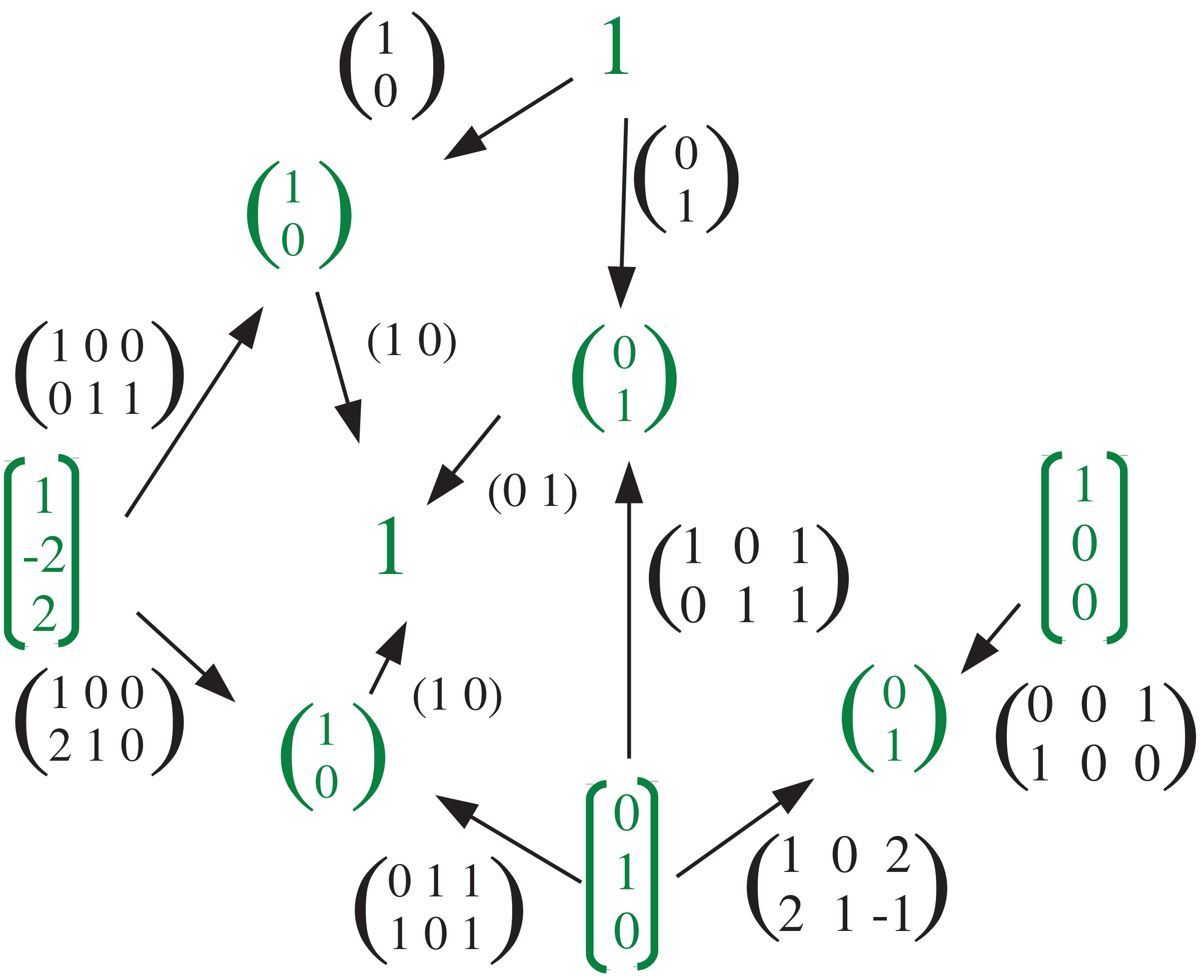
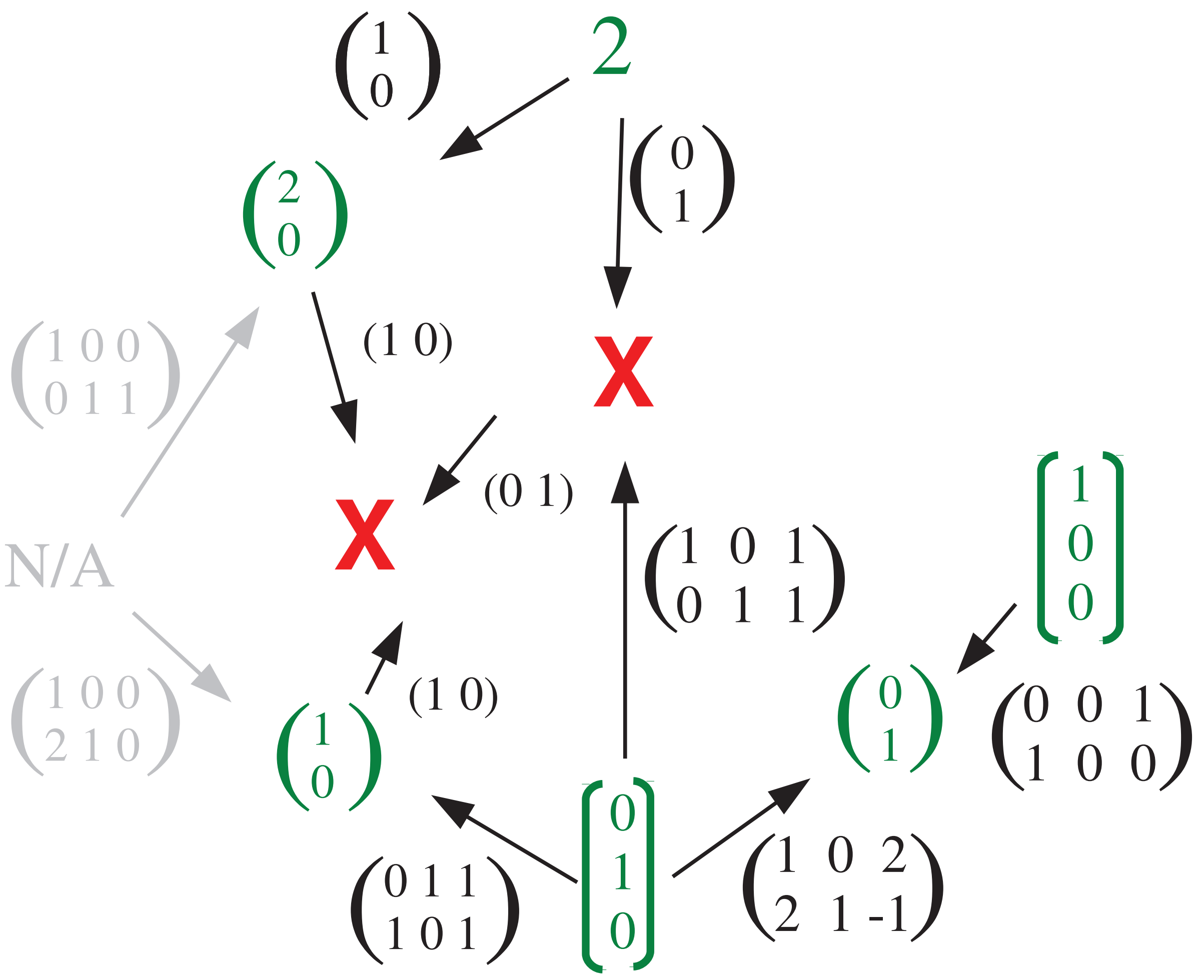
A *global section* is ...

- An assignment of values from each of the stalks that is consistent with the restrictions



Some sections are only local

- They might not be defined on all simplices or disagree with restrictions



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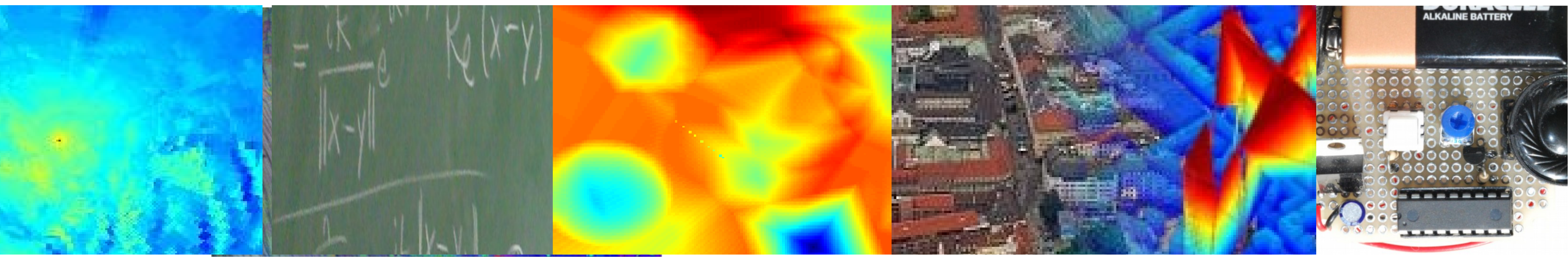
Sheaf Cohomology and its Interpretation



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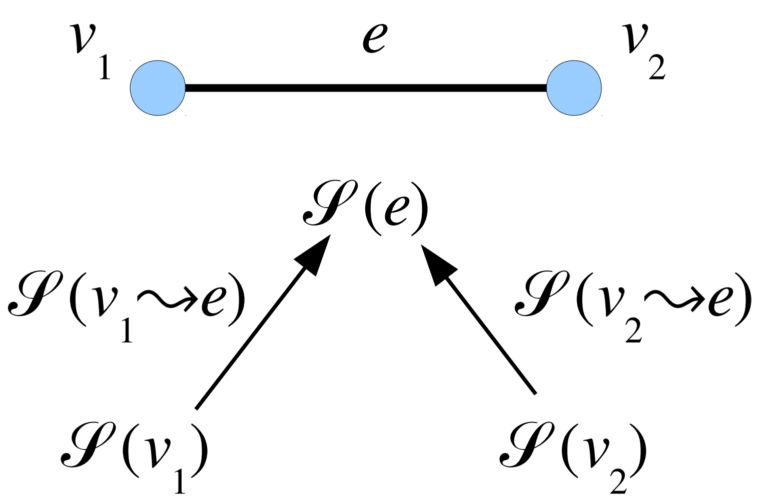
SIMPLEX Program

Global sections, revisited

- The space of global sections alone is insufficient to detect redundancy or possible faults, but another invariant works
- It's based on the idea that we can rewrite the basic condition(s) for a global section s of a sheaf $\mathscr{S}$



$$\mathscr{S}(v_1 \rightsquigarrow e)\, s(v_1) = \mathscr{S}(v_2 \rightsquigarrow e)\, s(v_2)$$

$$+\,\mathscr{S}(v_1 \rightsquigarrow e)\, s(v_1) - \mathscr{S}(v_2 \rightsquigarrow e)\, s(v_2) = 0$$

$$-\,\mathscr{S}(v_1 \rightsquigarrow e)\, s(v_1) + \mathscr{S}(v_2 \rightsquigarrow e)\, s(v_2) = 0$$

($\mathscr{S}(a \rightsquigarrow b)$ is the restriction map connecting cell a to a cell b in a sheaf $\mathscr{S}$)

Global sections, revisited

- The space of global sections alone is insufficient to detect redundancy or possible faults, but another invariant works
- It's based on the idea that we can rewrite the basic condition(s) for a global section s of a sheaf $\mathscr{S}$

$$\mathscr{S}(v_1 \rightsquigarrow e)\, s(v_1) = \mathscr{S}(v_2 \rightsquigarrow e)\, s(v_2)$$

$$\begin{pmatrix} +\mathscr{S}(v_1 \rightsquigarrow e) & -\mathscr{S}(v_2 \rightsquigarrow e) \end{pmatrix} \begin{pmatrix} s(v_1) \\ s(v_2) \end{pmatrix} = 0$$

$$+\mathscr{S}(v_1 \rightsquigarrow e)\, s(v_1) - \mathscr{S}(v_2 \rightsquigarrow e)\, s(v_2) = 0$$

$$-\mathscr{S}(v_1 \rightsquigarrow e)\, s(v_1) + \mathscr{S}(v_2 \rightsquigarrow e)\, s(v_2) = 0$$

($\mathscr{S}(a \rightsquigarrow b)$ is the restriction map connecting cell a to a cell b in a sheaf $\mathscr{S}$)

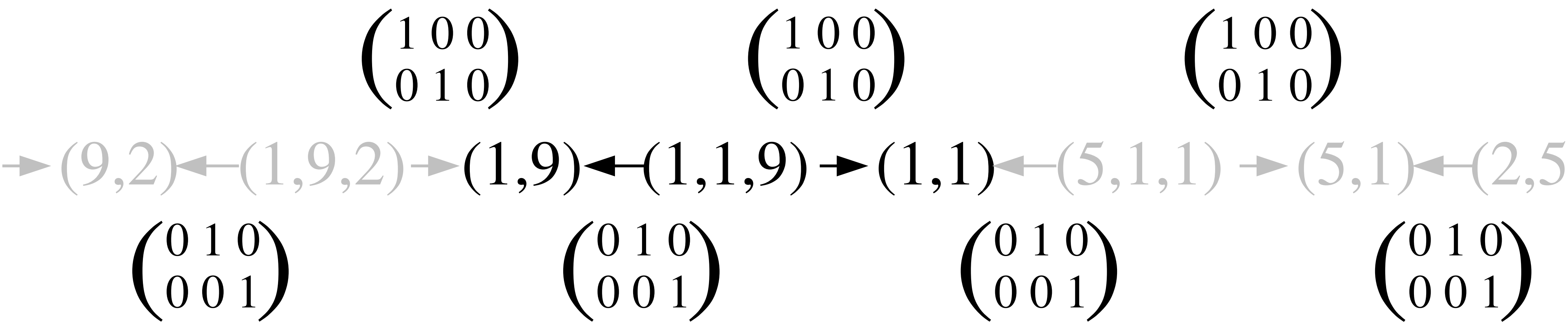
Recall: A queue as a sheaf

- Contents of the shift register at each timestep
- $N = 3$ shown

$$\rightarrow \mathbb{R}^2 \xleftarrow{\begin{pmatrix}0&1&0\\0&0&1\end{pmatrix}} \mathbb{R}^3 \xrightarrow{\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}} \mathbb{R}^2 \xleftarrow{\begin{pmatrix}0&1&0\\0&0&1\end{pmatrix}} \mathbb{R}^3 \xrightarrow{\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}} \mathbb{R}^2 \xleftarrow{\begin{pmatrix}0&1&0\\0&0&1\end{pmatrix}} \mathbb{R}^3 \xrightarrow{\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}} \mathbb{R}^2 \xleftarrow{\begin{pmatrix}0&1&0\\0&0&1\end{pmatrix}} \mathbb{R}$$

Recall: A single timestep

- Contents of the shift register at each timestep
- $N = 3$ shown



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Rewriting using matrices

- Same section, but the condition for verifying that it's a section is now written linear algebraically

$$(1,9,2) \xrightarrow{\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}} (1,9) \xleftarrow[\begin{pmatrix}0&1&0\\0&0&1\end{pmatrix}]{} (1,1,9) \xrightarrow{\begin{pmatrix}1&0&0\\0&1&0\end{pmatrix}} (1,1) \xleftarrow[\begin{pmatrix}0&1&0\\0&0&1\end{pmatrix}]{} (5,1,1)$$

$$\Longrightarrow \begin{pmatrix} 1&0&0 & 0&-1&0 & 0&0&0 \\ 0&1&0 & 0&0&-1 & 0&0&0 \\ 0&0&0 & 1&0&0 & 0&-1&0 \\ 0&0&0 & 0&1&0 & 0&0&-1 \end{pmatrix} \begin{pmatrix} 1\\9\\2\\1\\1\\9\\5\\1\\1 \end{pmatrix} = \begin{pmatrix} 0\\0\\0\\0 \end{pmatrix}$$

The *cochain complex*

- Motivation: Sections being in the kernel of matrix suggests a higher dimensional construction exists!
- Goal: build the *cochain complex* for a sheaf $\mathscr{S}$

$$; \mathscr{S}) \xrightarrow{d^{k-1}} C^k(X; \mathscr{S}) \xrightarrow{d^k} C^{k+1}(X; \mathscr{S}) \xrightarrow{d^{k+1}} C^{k+2}(X;$$

- From this, *sheaf cohomology* will be defined as

$$H^k(X; \mathscr{S}) = \ker d^k \,/\, \text{image } d^{k-1}$$

much the same as homology (but the chain complex goes up in dimension instead of down)

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Generalizing up in dimension

- Global sections lie in the kernel of a particular matrix
- We gather the domain and range from stalks over vertices and edges... These are the *cochain spaces*

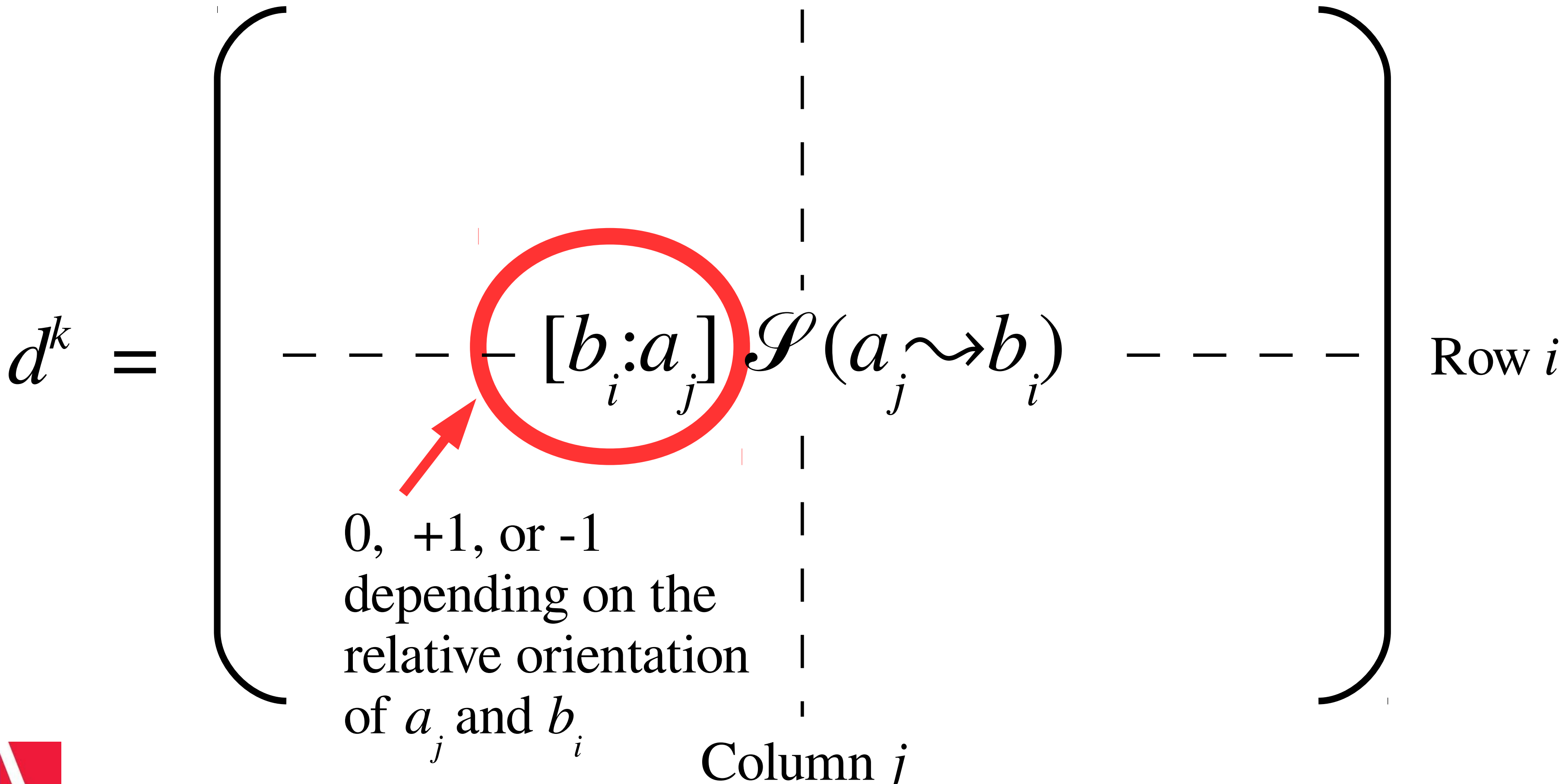
$$C^k(X;\mathscr{S}) = \bigoplus_{a \text{ is a } k\text{-simplex}} \mathscr{S}(a)$$

- An element of $C^k(X;\mathscr{S})$ is called a *cochain*, and specifies a datum from the stalk at each k-simplex

(The *direct sum* operator $\bigoplus$ forms a new vector space by concatenating the bases of its operands)

The cochain complex

- The *coboundary map* $d^k : C^k(X; \mathscr{S}) \rightarrow C^{k+1}(X; \mathscr{S})$ is given by the block matrix



The cochain complex

- We've obtained the *cochain complex*

$$\cdots ; \mathcal{S}) \xrightarrow{d^{k-1}} C^k(X; \mathcal{S}) \xrightarrow{d^k} C^{k+1}(X; \mathcal{S}) \xrightarrow{d^{k+1}} C^{k+2}(X; \cdots$$

- *Cohomology* is defined as

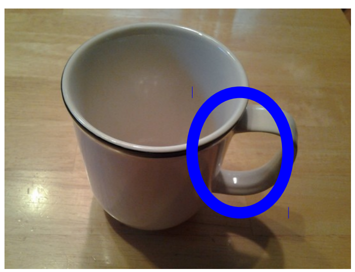
$$H^k(X; \mathcal{S}) = \ker d^k \, / \, \text{image } d^{k-1}$$

All the cochains that are consistent in dimension k ...

… that weren't already present in dimension k - 1

Cohomology facts

- $H^0(X; \mathscr{S})$ is the space of global sections of $\mathscr{S}$
- $H^1(X; \mathscr{S})$ usually has to do with oriented, non-collapsible **data** loops



Nontrivial $H^1(X; \mathbb{Z})$

- $H^k(X; \mathscr{S})$ is a functor: sheaf morphisms induce linear maps between cohomology spaces

Cohomology versus homology

Homologies of different chain complexes:

- Chain complex: simplices and their boundaries

$$\longrightarrow C_{k+1}(X) \xrightarrow{\partial_{k+1}} C_k(X) \xrightarrow{\partial_k} C_{k-1}(X) \xrightarrow{\partial_{k-1}}$$

- Transposing the boundary maps yields the *cochain complex*: functions on simplices

$$\xleftarrow{\partial^T_{k+2}} C_{k+1}(X) \xleftarrow{\partial^T_{k+1}} C_k(X) \xleftarrow{\partial^T_k} C_{k-1}(X) \xleftarrow{\partial^T_{k-1}}$$

- With $\mathbb{R}$ linear algebra, homology* of both of these carry identical information for a wide class of spaces

* we call the homology of a cochain complex *cohomology*

Cohomology versus homology

Homologies of different chain complexes:

- Transposing the boundary maps yields the *cochain complex*: functions on simplices

$$\xleftarrow{\partial^{T}_{k+2}} C_{k+1}(X) \xleftarrow{\partial^{T}_{k+1}} C_{k}(X) \xleftarrow{\partial^{T}_{k}} C_{k-1}(X) \xleftarrow{\partial^{T}_{k-1}}$$

The *co*boundary maps work like discrete derivatives and compute differences between functions on higher dimensional simplices

Sheaf cohomology versus homology

Homologies of different chain complexes:

- Transposing the boundary maps yields the *cochain complex*: functions on simplices

$$\xleftarrow{\partial^T_{k+2}} C_{k+1}(X) \xleftarrow{\partial^T_{k+1}} C_k(X) \xleftarrow{\partial^T_k} C_{k-1}(X) \xleftarrow{\partial^T_{k-1}}$$

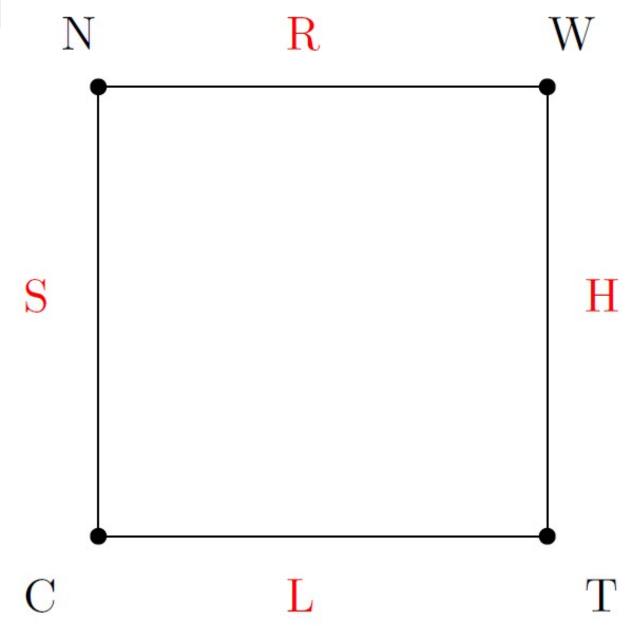
- *Sheaf cochain complex*: also functions on simplices, but they are generalized!

$$X; \mathscr{S}) \xleftarrow{d^{k+1}} C^{k+1}(X; \mathscr{S}) \xleftarrow{d^k} C^k(X; \mathscr{S}) \xleftarrow{d^{k-1}} C^{k-1}(X;$$

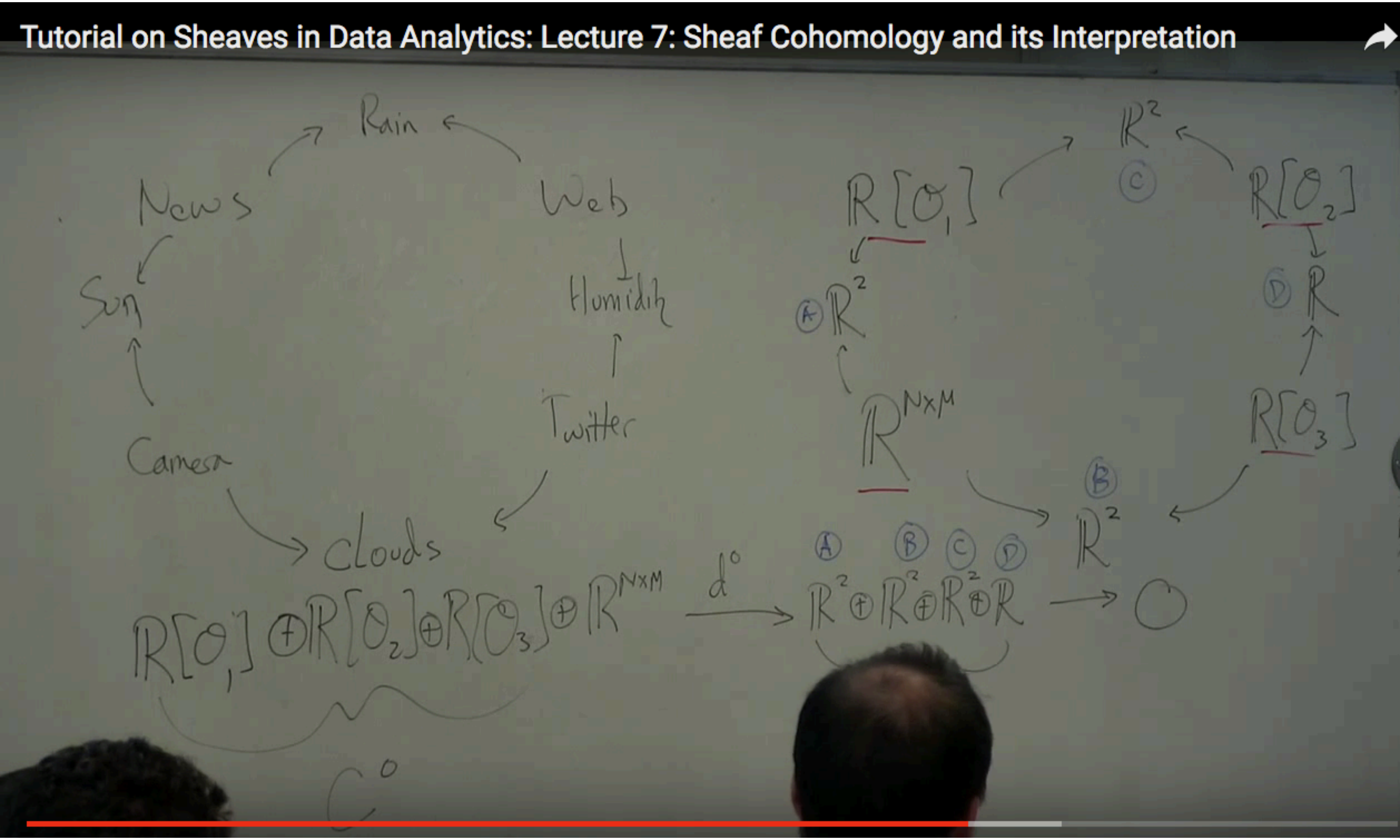
"Weather Loop" a simple model

Sensors/ Questions	Rain? (R)	Humidity % (H)	Clouds? (L)	Sun? (S)
News (N)	X			X
Weather Website (W)	X	X		
Rooftop Camera (C)			X	X
Twitter (T)		X	X	

Make simplicial complex



Question: Can misleading globalized information be detected?

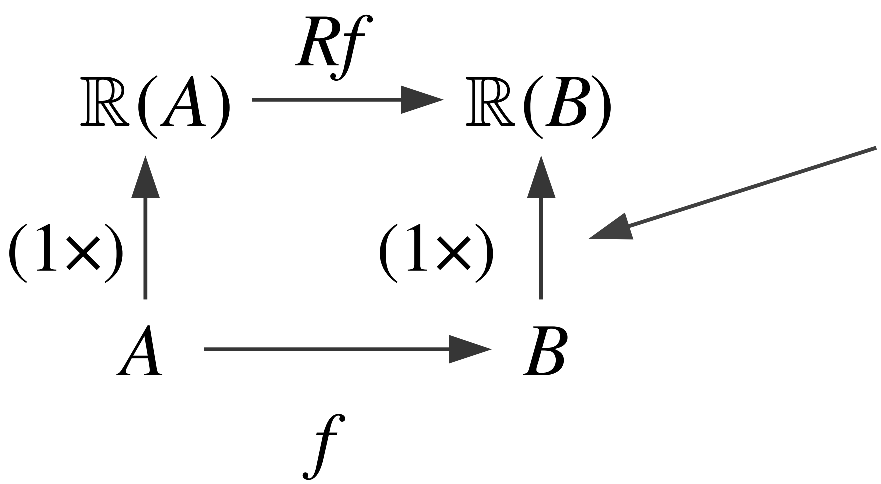


https://youtu.be/dAfVrTDFcs4?list=PLSekr_gm4hWLvFtJX0WUueVO65uhvBPrA

Lifting functions into linear maps

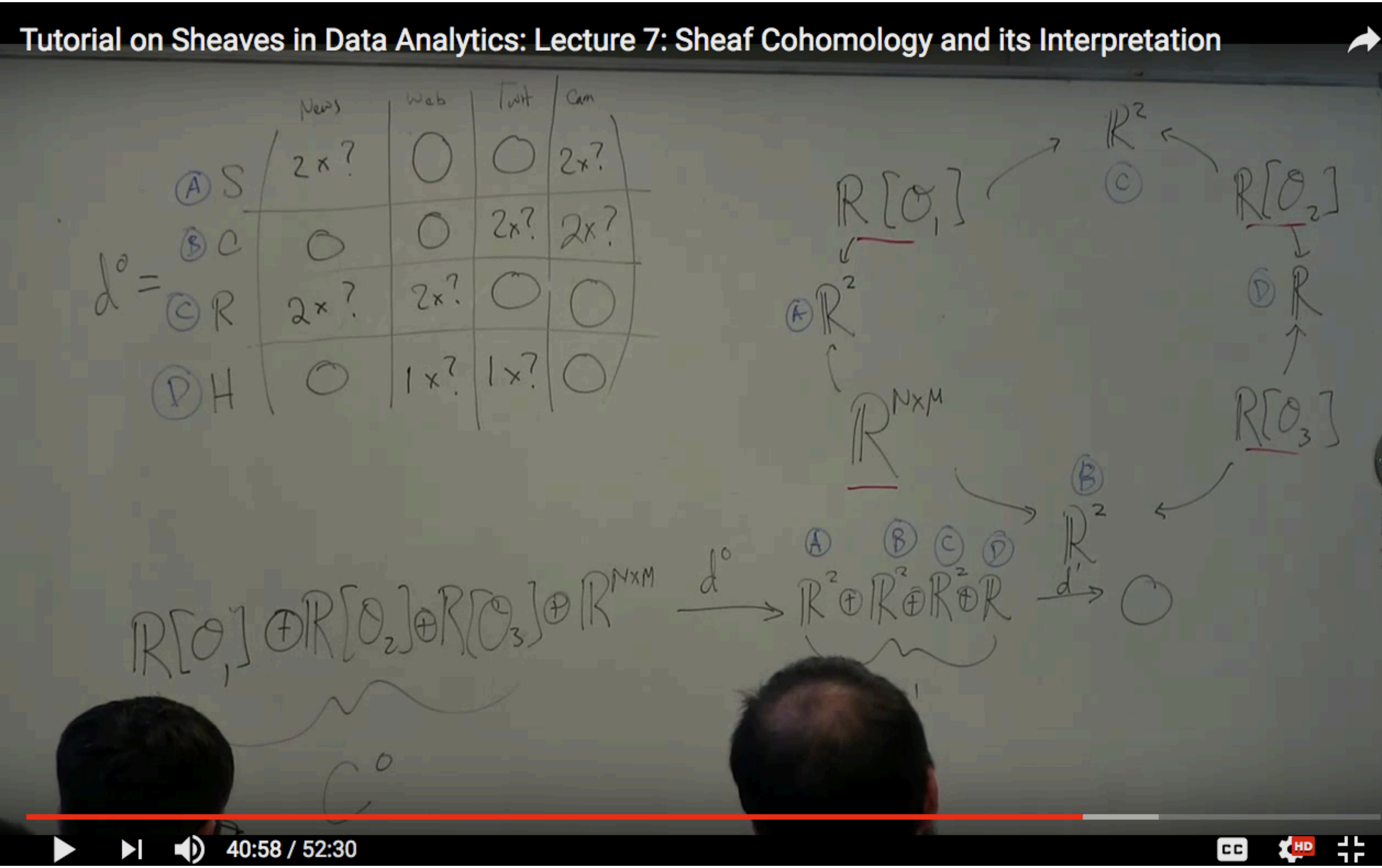
Consider any function between sets $f: A \to B$

- Let $\mathbb{R}(A)$ be the vector space generated by A
  - The basis of $\mathbb{R}(A)$ is the set of elements of A
- Then f lifts uniquely to a linear map Rf



Notice that generally we cannot recover a unique element of B from $\mathbb{R}(B)$.

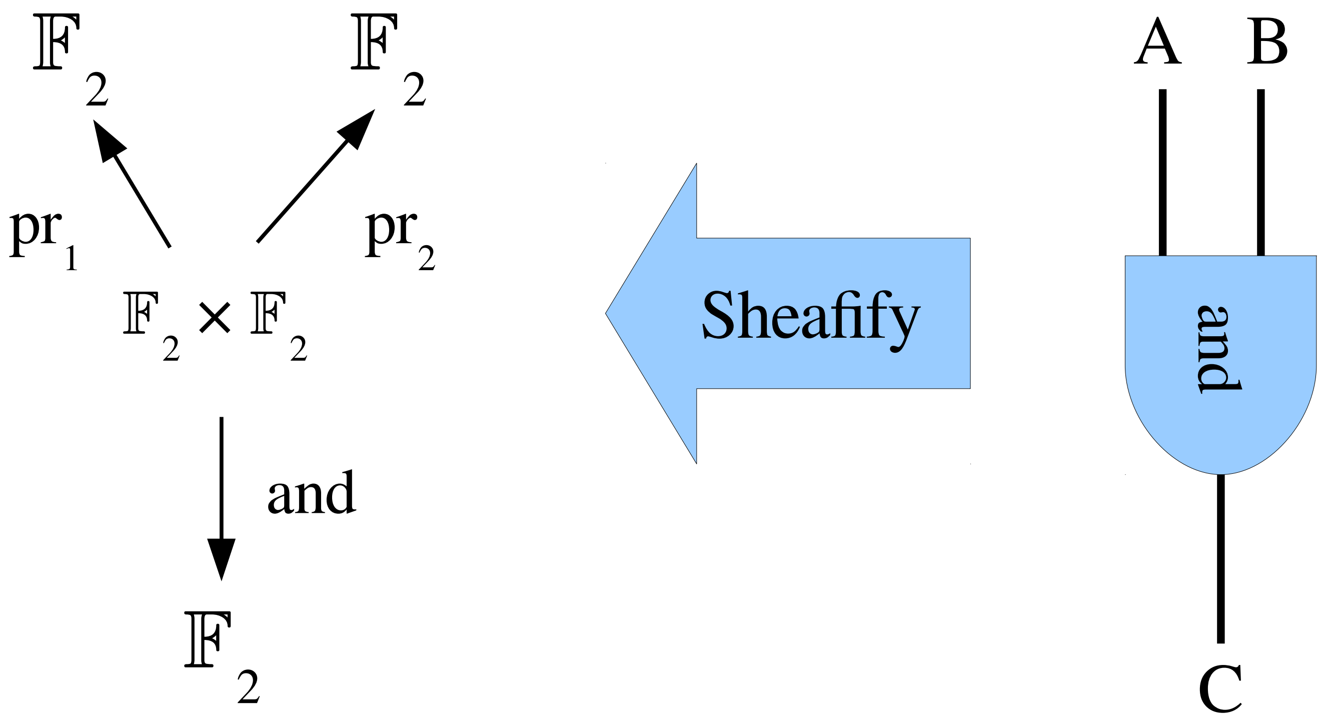
But we can if we've used $Rf \circ (1\times)$



https://youtu.be/dAfVrTDFcs4?list=PLSekr_gm4hWLvFtJX0WUueVO65uhvBPrA

Switching sheaves

- It's possible to construct a sheaf that represents the truth table of a logic circuit
- Each vertex is a logic gate, each edge is a wire



A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

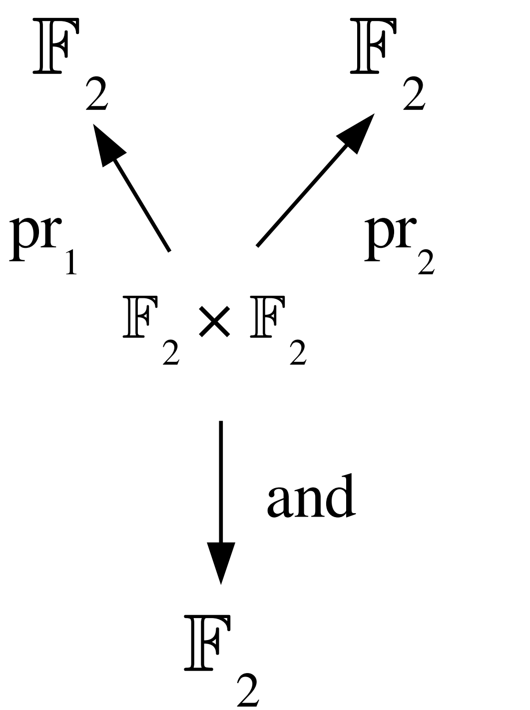
Quiescent* logic sheaf

Logic circuit

*Quiescent = steady state, pr_n = projection onto *n*th component

Switching sheaves

- Vectorify **everything** about a quiescent logic sheaf, and you obtain a *switching sheaf*



Quiescent* logic sheaf

$\mathbb{F}_2[\mathbb{F}_2 \times \mathbb{F}_2]$

=

A vector space whose basis is the set of ordered pairs

=

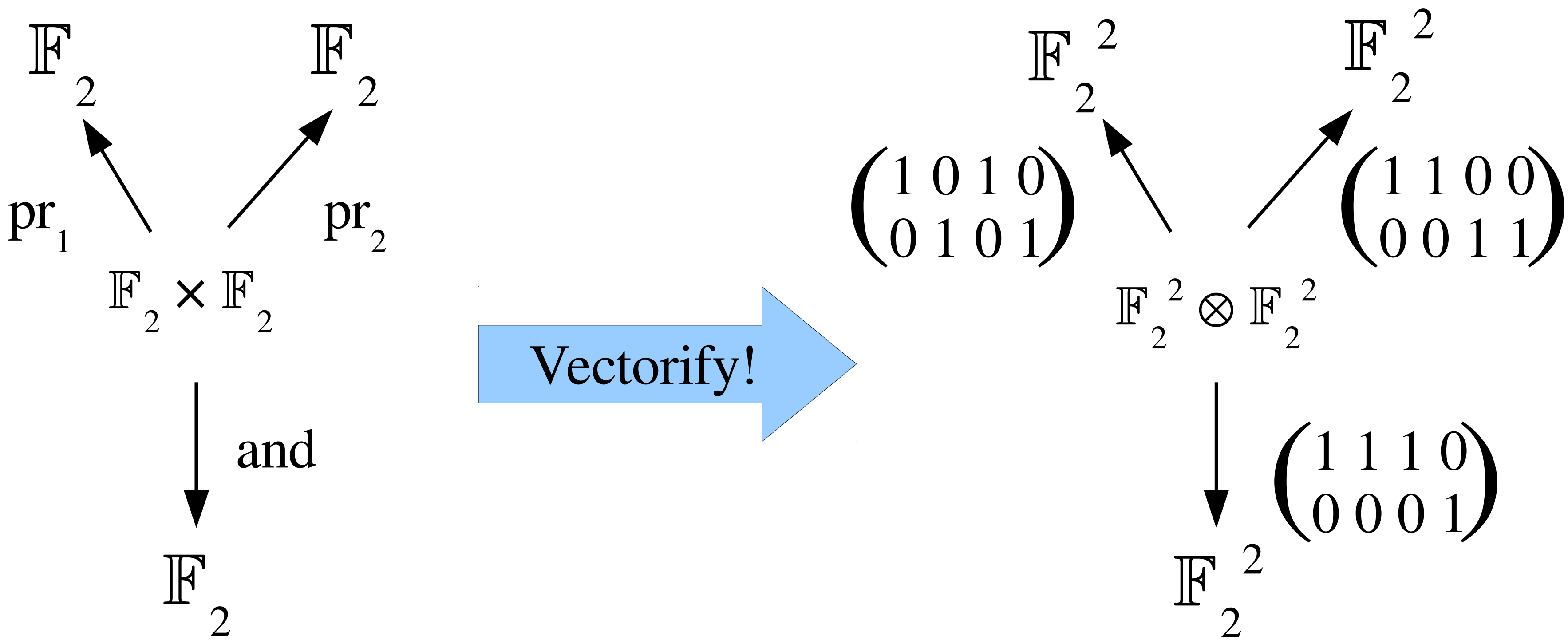
$\mathbb{F}_2^{\,2} \otimes \mathbb{F}_2^{\,2}$

$\otimes$ = Tensor product

*Quiescent = steady state, pr_n = projection onto *n*th component

Switching sheaves

- Vectorify **everything** about a quiescent logic sheaf, and you obtain a *switching sheaf*

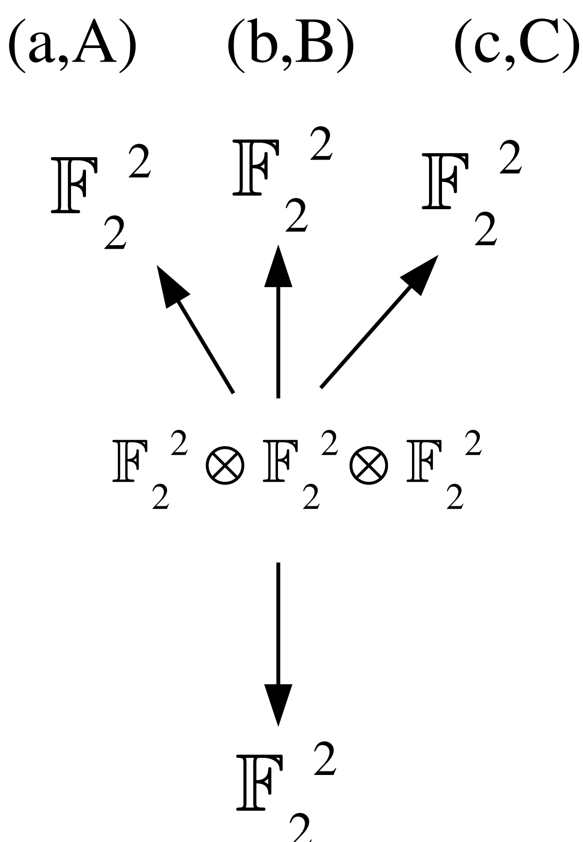


Quiescent* logic sheaf | Switching sheaf

*Quiescent = steady state, pr_n = projection onto *n*th component

Global sections of switching sheaves

- In the case of a 3 input gate, the global sections are spanned by **all simultaneous combinations** of inputs



$a \otimes b \otimes c$

$a \otimes b \otimes C$

$a \otimes B \otimes c$

$a \otimes B \otimes C$

$A \otimes b \otimes c$

$A \otimes b \otimes C$

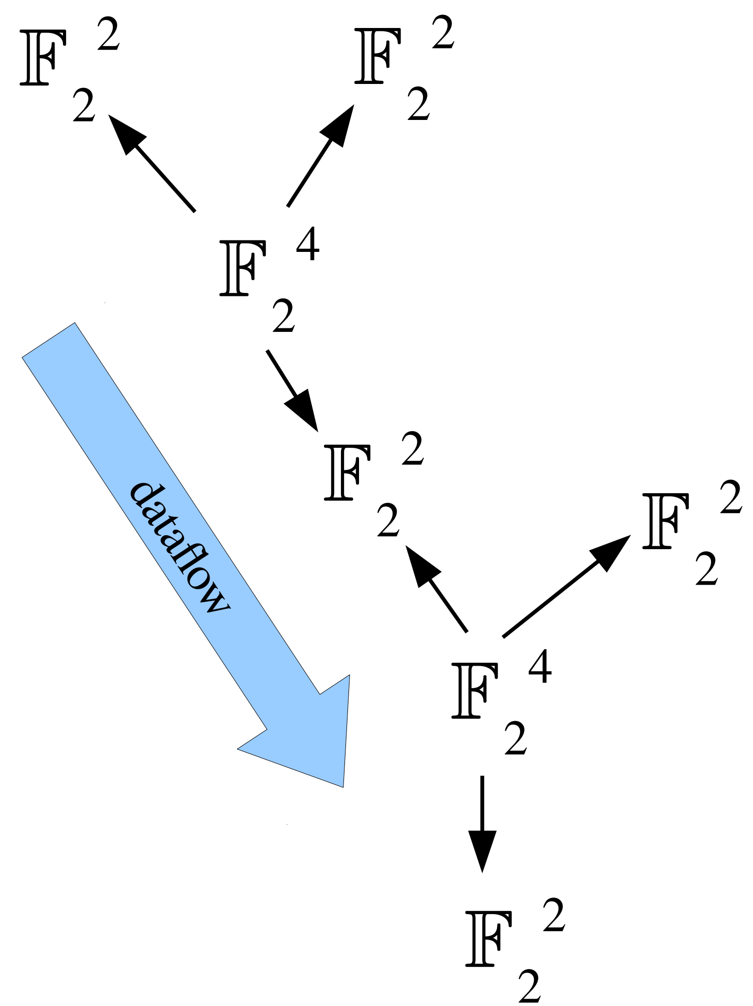
$A \otimes B \otimes c$

$A \otimes B \otimes C$

$2^8 = 256$ sections in total

Global sections of switching sheaves

- When we instead consider a logically equivalent circuit, the situation changes
- Global sections consist of simultaneous inputs to each gate, but consistency is checked via tensor contractions
- There is an inherent model of uncertainty



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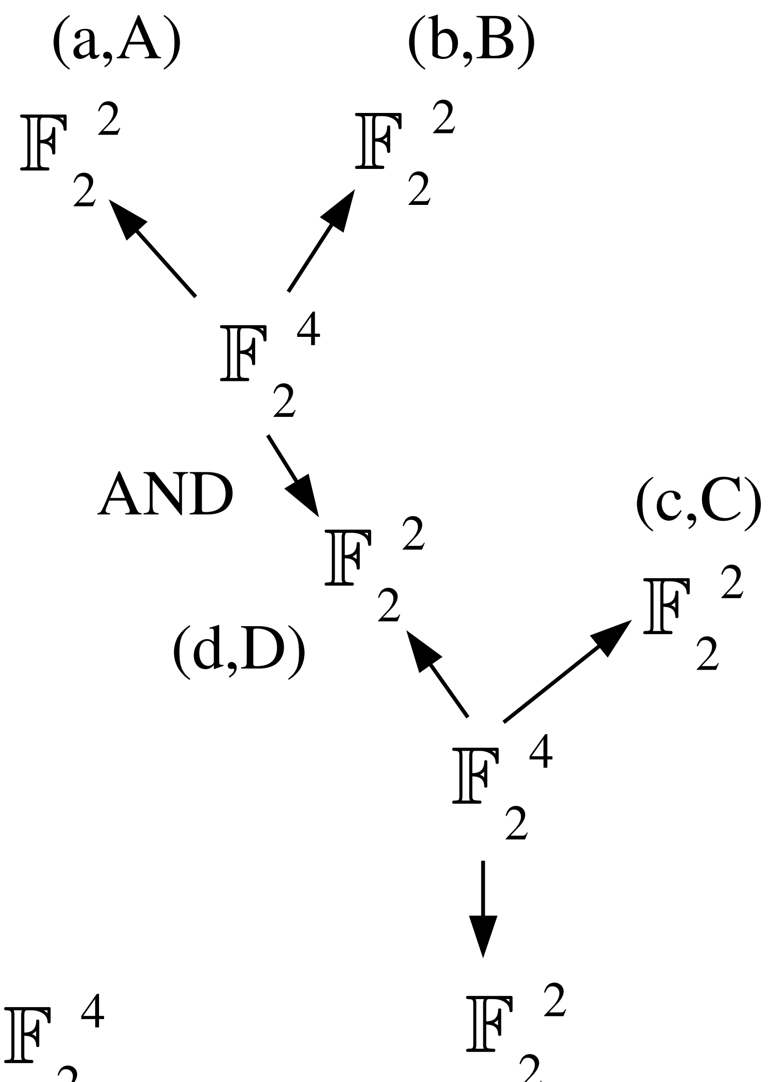
Global sections of switching sheaves

- The space of global sections is now 6 dimensional – some sections were lost!

$$a \otimes b + c \otimes d$$
$$a \otimes b + C \otimes d$$
$$a \otimes B + c \otimes d$$
$$A \otimes b + c \otimes d$$
$$A \otimes B + c \otimes D$$
$$A \otimes B + C \otimes D$$

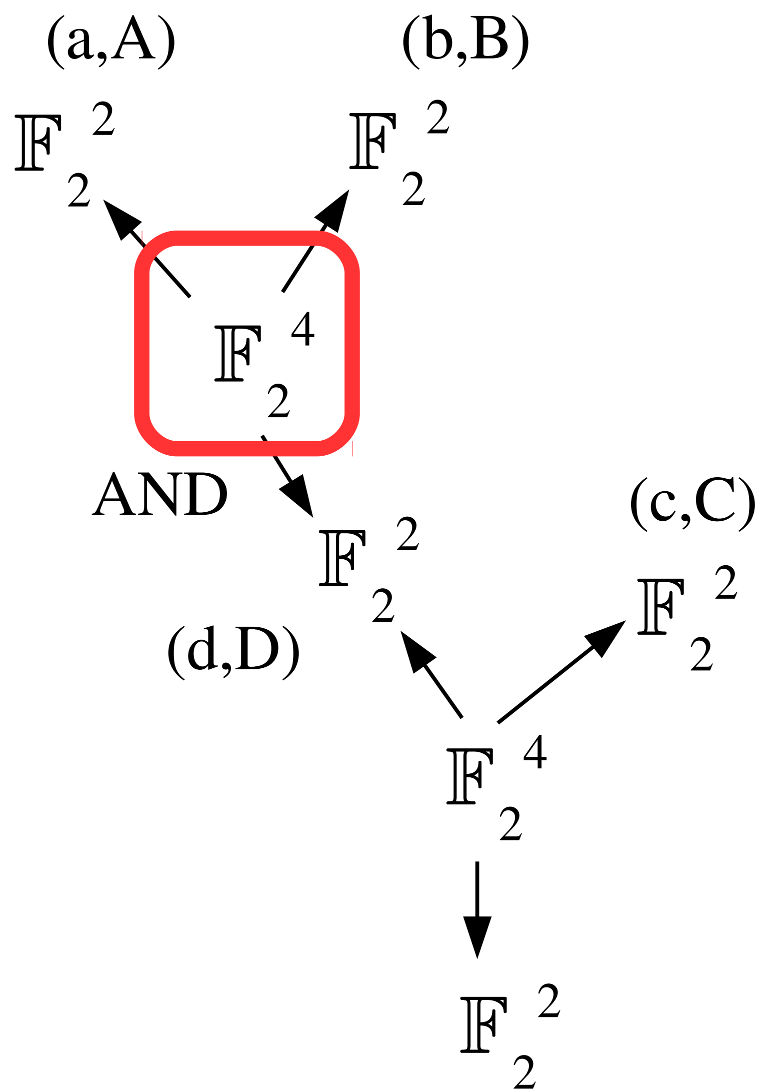


Recall that the space of global sections is a subspace of $\mathbb{F}_2^4 \oplus \mathbb{F}_2^4$

Global sections of switching sheaves

- The space of global sections is now 6 dimensional – some sections were lost!

$$a \otimes b + c \otimes d$$
$$a \otimes b + C \otimes d$$
$$a \otimes B + c \otimes d$$
$$A \otimes b + c \otimes d$$
$$A \otimes B + c \otimes D$$
$$A \otimes B + C \otimes D$$



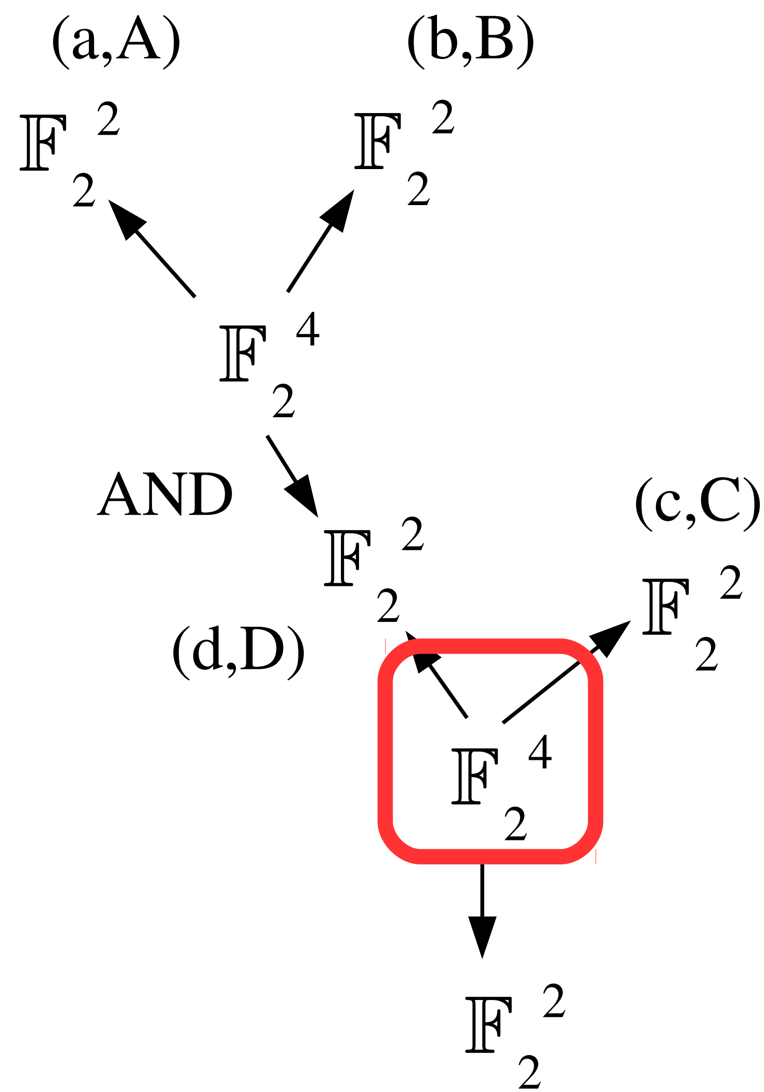
- All local sections on the upstream gate are represented

Global sections of switching sheaves

- The space of global sections is now 6 dimensional – some sections were lost!

$$a \otimes b + c \otimes d$$
$$a \otimes b + C \otimes d$$
$$a \otimes B + c \otimes d$$
$$A \otimes b + c \otimes d$$
$$A \otimes B + c \otimes D$$
$$A \otimes B + C \otimes D$$

- All local sections supported on the downstream gate are there too



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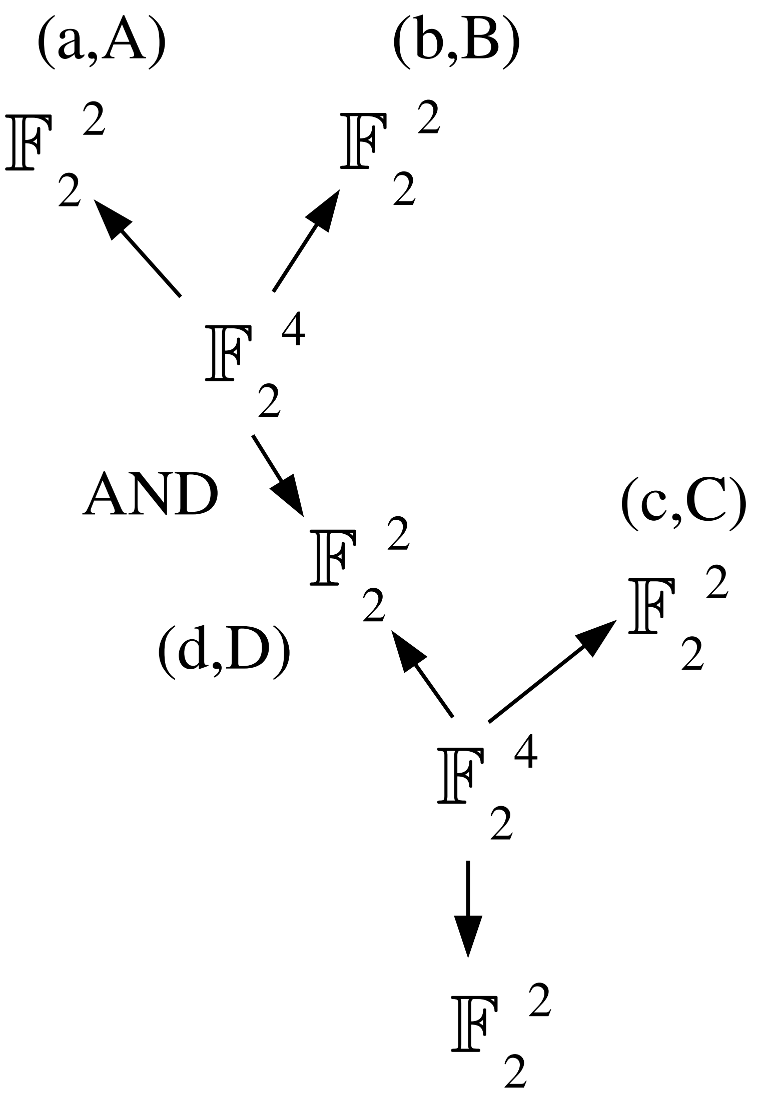
Global sections of switching sheaves

No quiescent logic states were actually lost, but the sections of this sheaf represent sets of simultaneous data at each gate that might be **in transition**!



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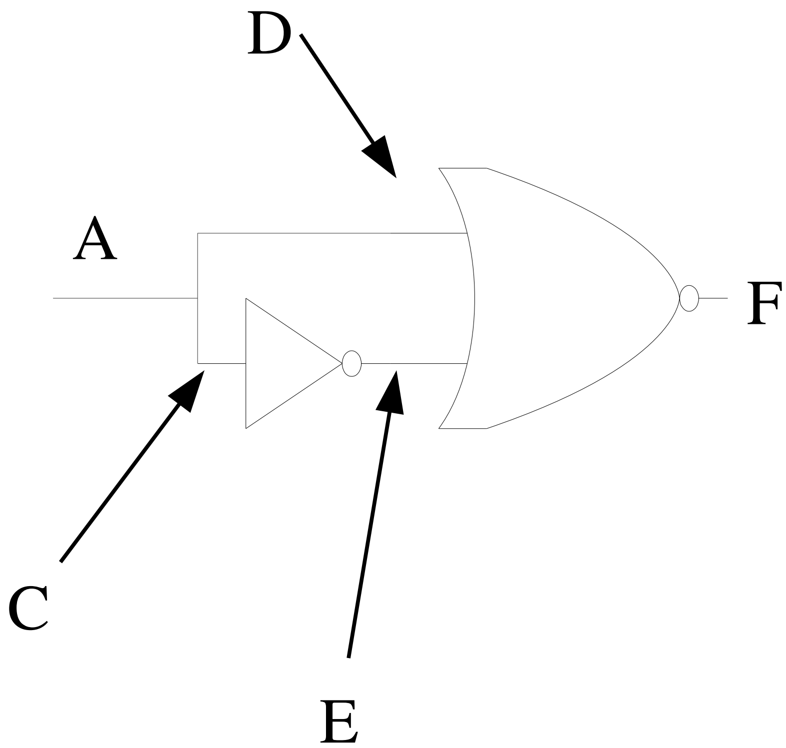
Higher cohomology spaces

- Switching sheaves are written over 1-dimensional spaces, so they could have nontrivial 1-cohomology
- Nontrivial 1-cohomology classes consist of **directed loops that store data**
- Since we just found that logic value transitions are permitted, this means that 1-cohomology can detect glitches

Glitch generator: cohomology



$H^0(X;\mathscr{F})$ is generated by

A+C+D ⊗ e
a+c+d ⊗ E

A+a+C+c+d ⊗ e+D ⊗ E

Indication that there's a race condition possible → $H^1(X;\mathscr{F}) \cong \mathbb{Z}_2$

Hazard transition state

H^1 detects the race condition

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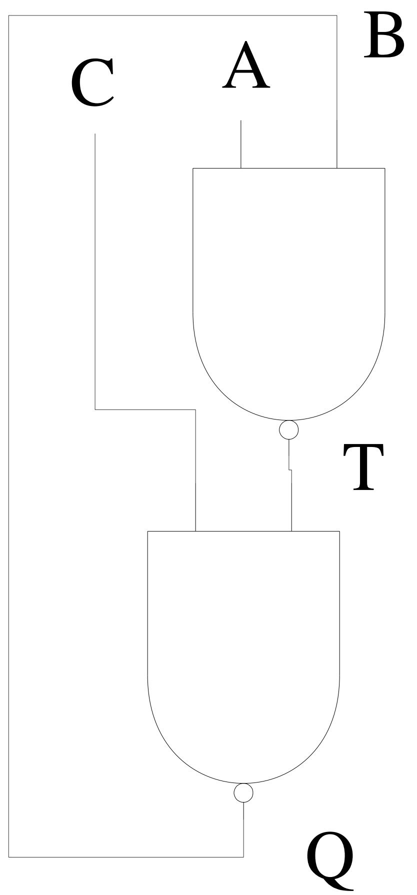
Example: flip-flop

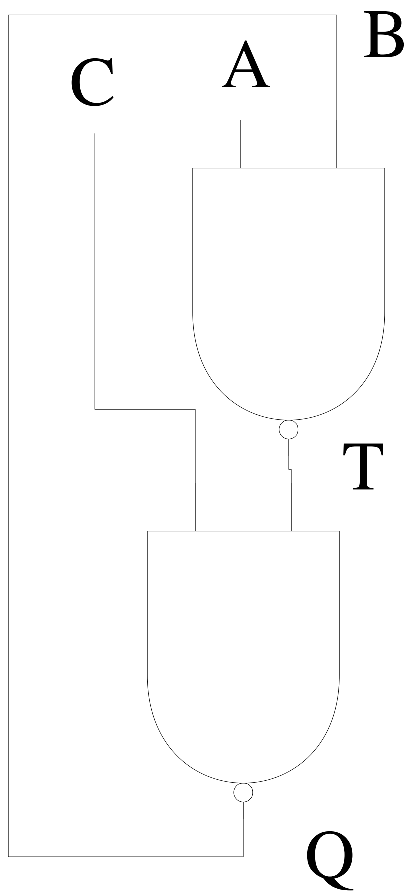


C	A	B	T	Q	
0	0	0	1	1	
0	0	1	1	1	Hazard!
0	1	0	1	1	
0	1	1	0	1	Set
1	0	0	1	0	Reset
1	0	1	1	0	
1	1	0	1	0	Hold
1	1	1	0	1	

Transition out of the hazard state to the hold state causes a race condition

This is what traditional analysis gives...
5 possible states

Flip-flop cohomology



$H^1(X;F) \cong \mathbb{Z}_2$ ← Race condition detected!

$H^0(X;F) \cong \mathbb{Z}_2^7$

Generated by:

$a \otimes B \otimes c$
$A \otimes B \otimes c$
$a \otimes b \otimes C$
$A \otimes b \otimes C$
$A \otimes B \otimes C$

States from the truth table

$a \otimes b \otimes c + a \otimes B \otimes C$
$a \otimes b \otimes c + A \otimes b \otimes c$

These states describe the possible transitions out of the hazard state – something that takes a bit more trouble to obtain traditionally

Bonus: Cosheaf homology

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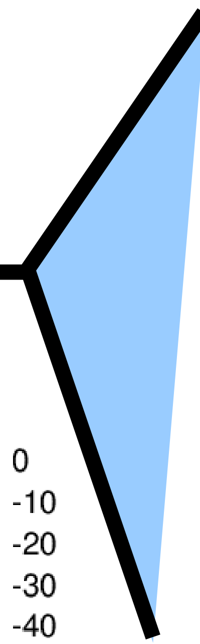
Cosheaf homology

- The globality of cosheaf sections concentrates in top dimension, which may vary over the base space
  - No particular degree of cosheaf homology holds global sections if the model varies in dimension
- But what **is** clear is that numerical instabilities can arise if certain nontrivial homology classes exist
  - These can obscure actual solutions, but can look "very real" resulting in confusion
  - There are many open questions…

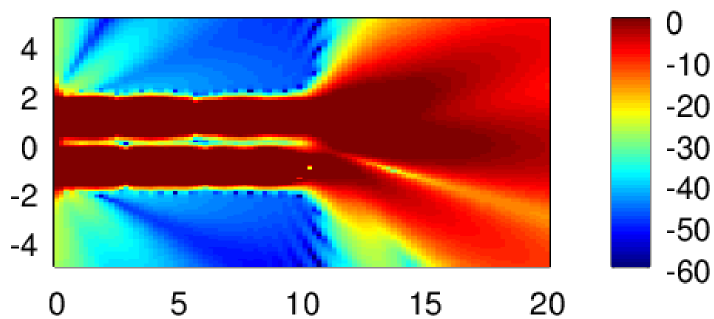
Wave propagation as cosheaf

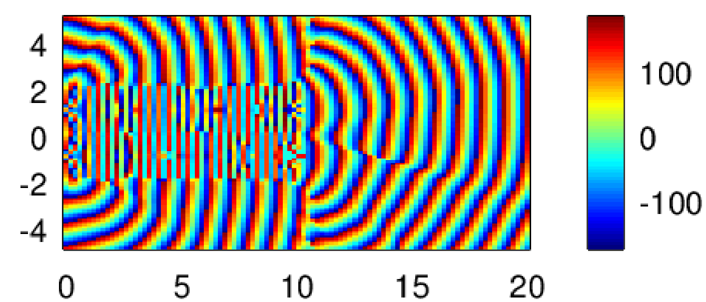
$\Delta u + k^2 u = 0$
with Dirichlet
boundary
conditions



Open aperture

Narrow feed channel





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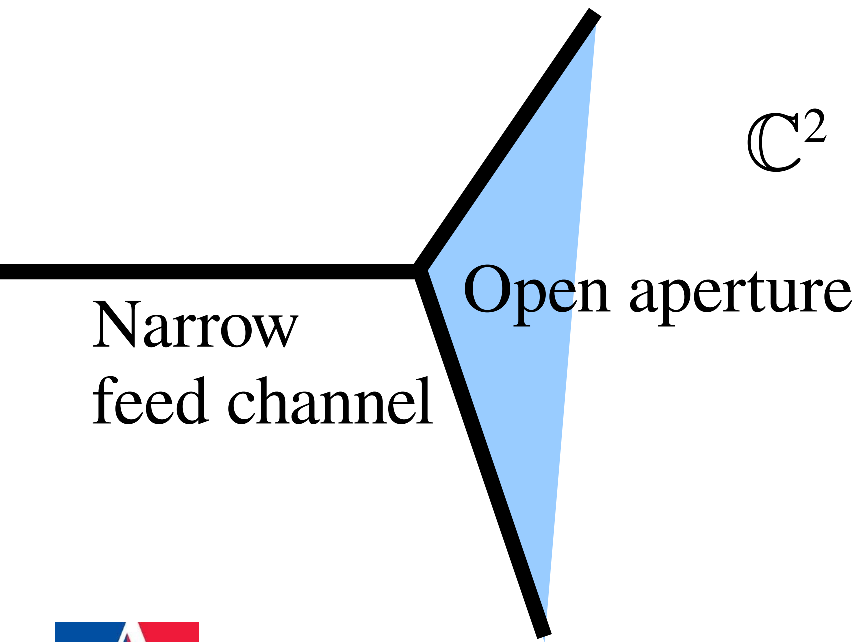
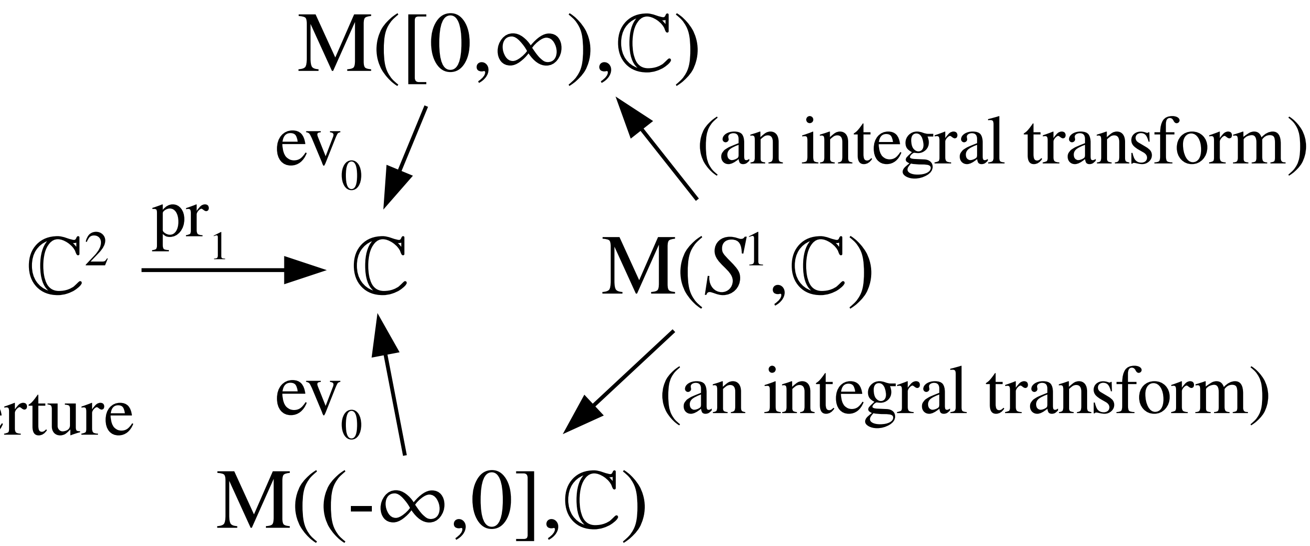
Wave propagation as cosheaf

- Solving $\Delta u + k^2 u = 0$ (single frequency wave propagation) on a cell complex with Dirichlet boundary conditions





$M(X,\mathbb{C})$ = Space of complex measures on X

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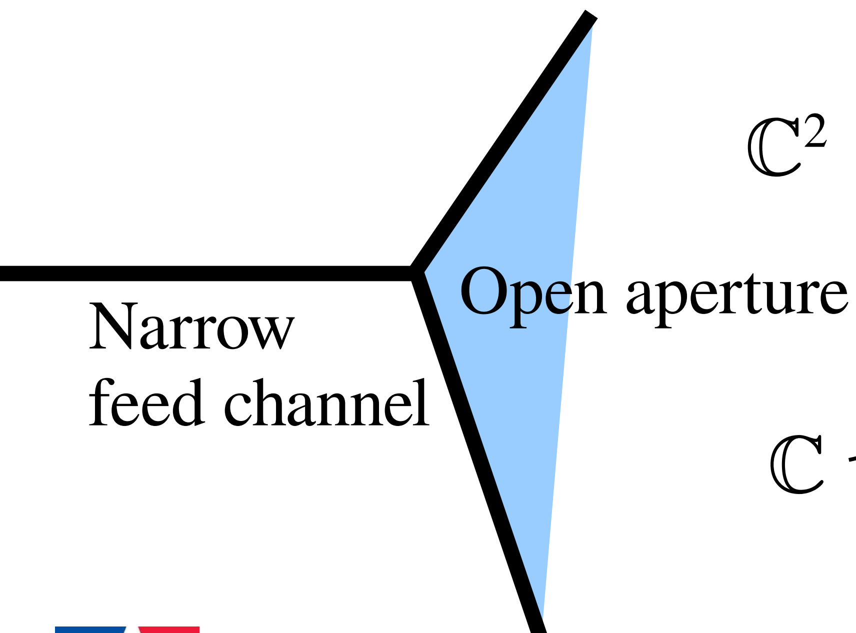
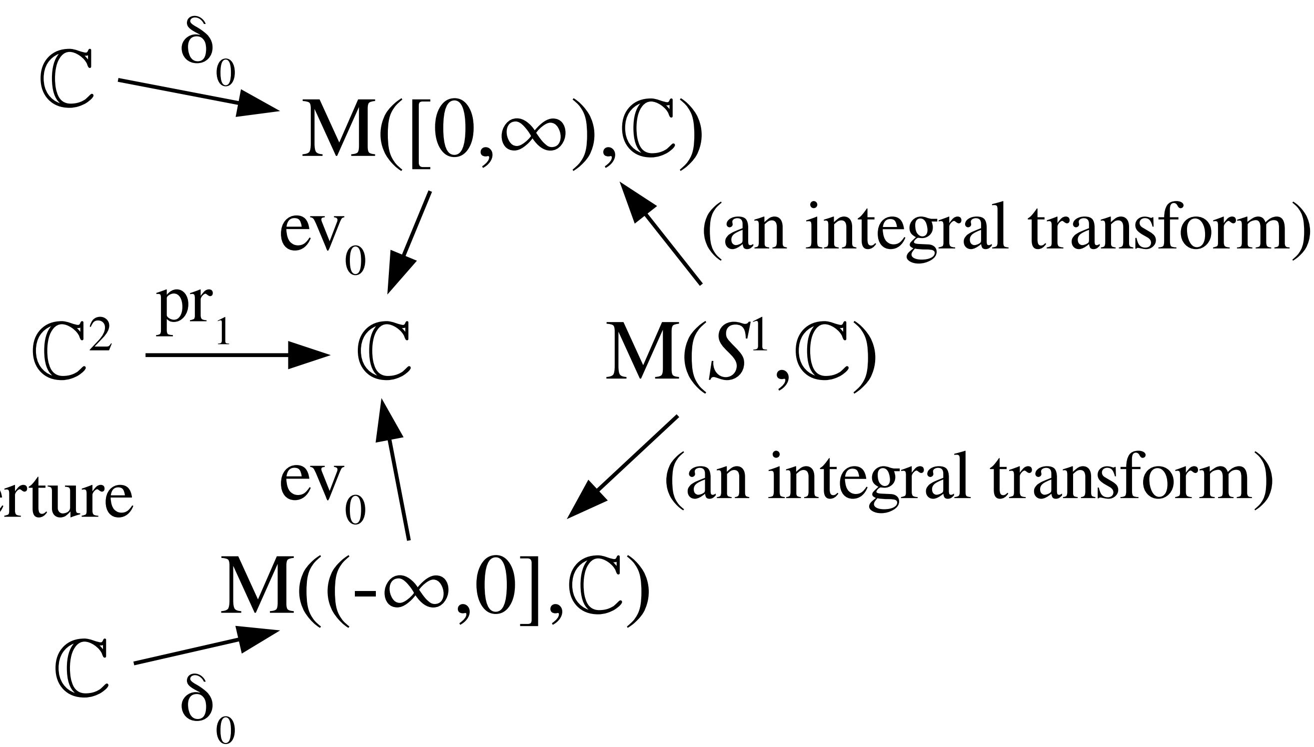
Wave propagation as cosheaf

- Solving $\Delta u + k^2 u = 0$ (single frequency wave propagation) on a cell complex with **Dirichlet boundary conditions**





$M(X,\mathbb{C})$ = Space of complex measures on X

Wave propagation cosheaf homology

- The global sections indeed get spread across dimension
- Here's the chain complex:

Dimension 2 | Dimension 1 | Dimension 0

$$M(S^1,\mathbb{C}) \oplus \mathbb{C} \oplus \mathbb{C} \rightarrow M((-\infty,0],\mathbb{C}) \oplus \mathbb{C}^2 \oplus M([0,\infty),\mathbb{C}) \longrightarrow \mathbb{C}$$

Global sections are parameterized by a subspace of these

Further reading...

- Louis Billera, "Homology of Smooth Splines: Generic Triangulations and a Conjecture of Strang," *Trans. Amer. Math. Soc.*, Vol. 310, No. 1, Nov 1998.
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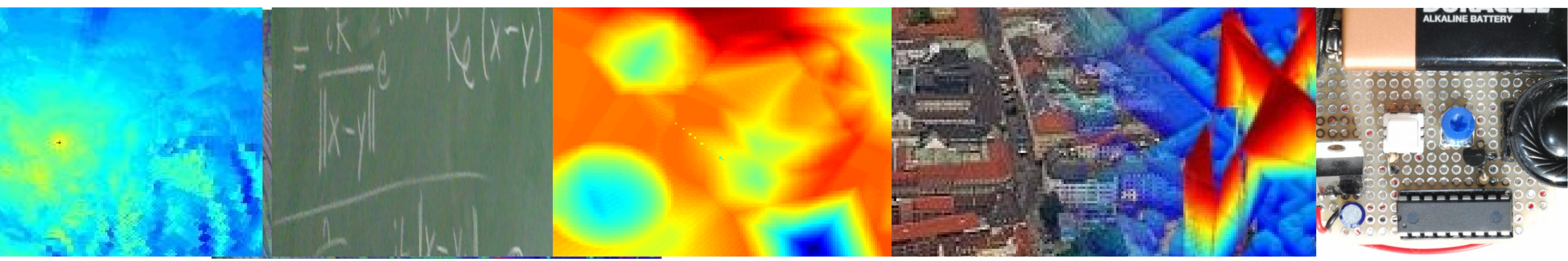
- Michael Robinson, "Inverse problems in geometric graphs using internal measurements," http://www.arxiv.org/abs/1008.2933
- Michael Robinson, "Asynchronous logic circuits and sheaf obstructions," *Electronic Notes in Theoretical Computer Science* (2012), pp. 159-177.
- Pierre Schapira, "Sheaf theory for partial differential equations," *Proc. Int. Congress Math.*, Kyoto, Japan, 1990.

How do we Deal with Noisy Data?



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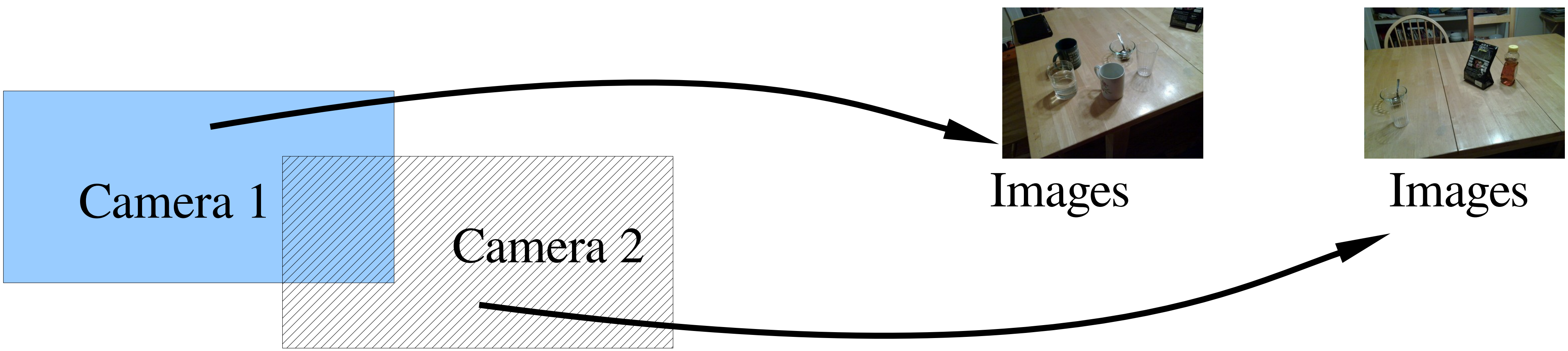
DARPA SIMPLEX Program

Inexact matching

- Sections of a sheaf are great: they globalize information across data sources
- But they seem to require **exact** matches between data sources, which is undesirable…



- What if instead we want matches that are approximate, to a certain tolerance?

Relaxing matching requirements

- Sections require exact matches...

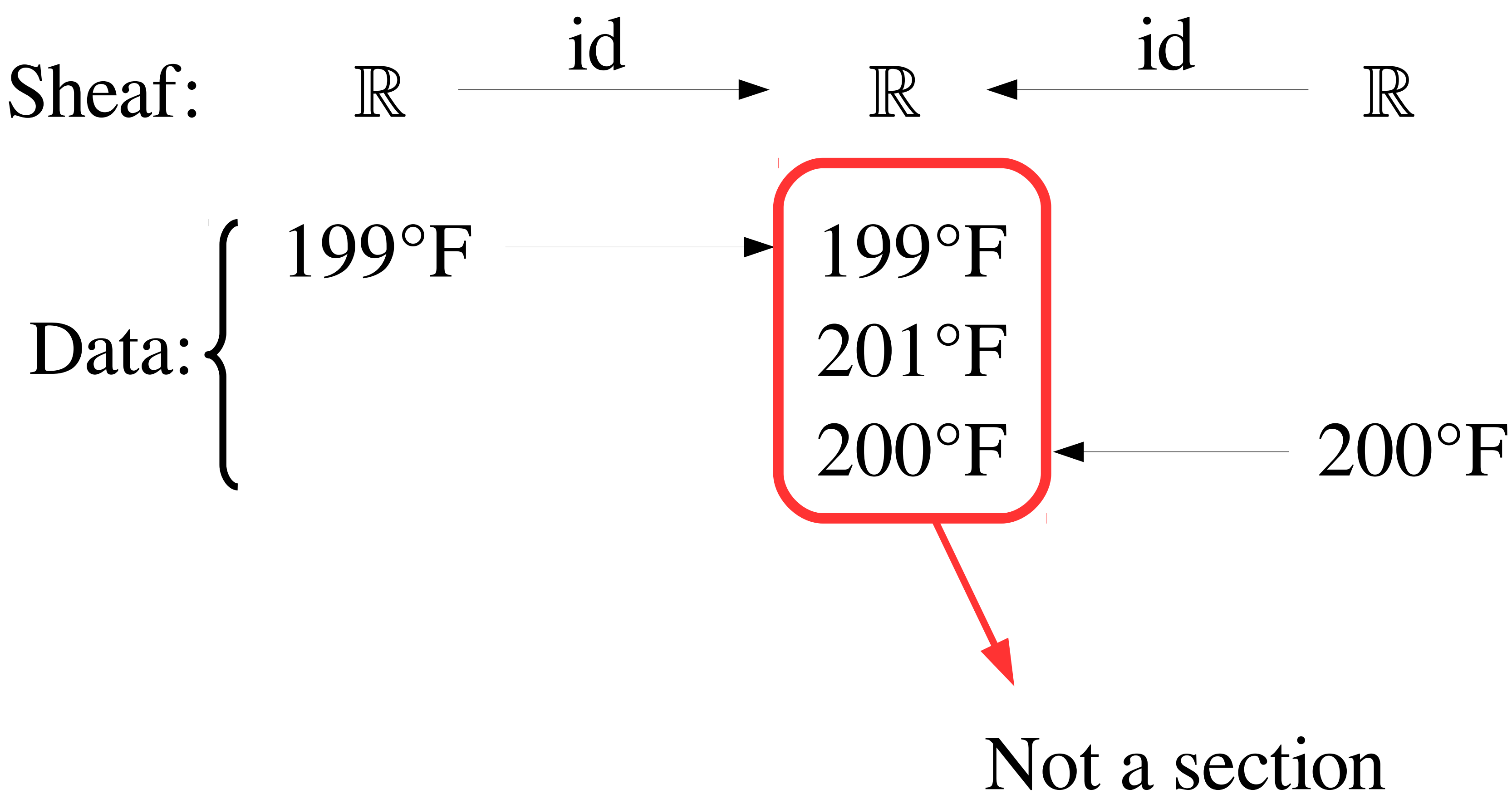
Sheaf: $\mathbb{R} \xrightarrow{\text{id}} \mathbb{R} \xleftarrow{\text{id}} \mathbb{R}$

Data:
200°F → 200°F
200°F
200°F ← 200°F

All equal → It's a section

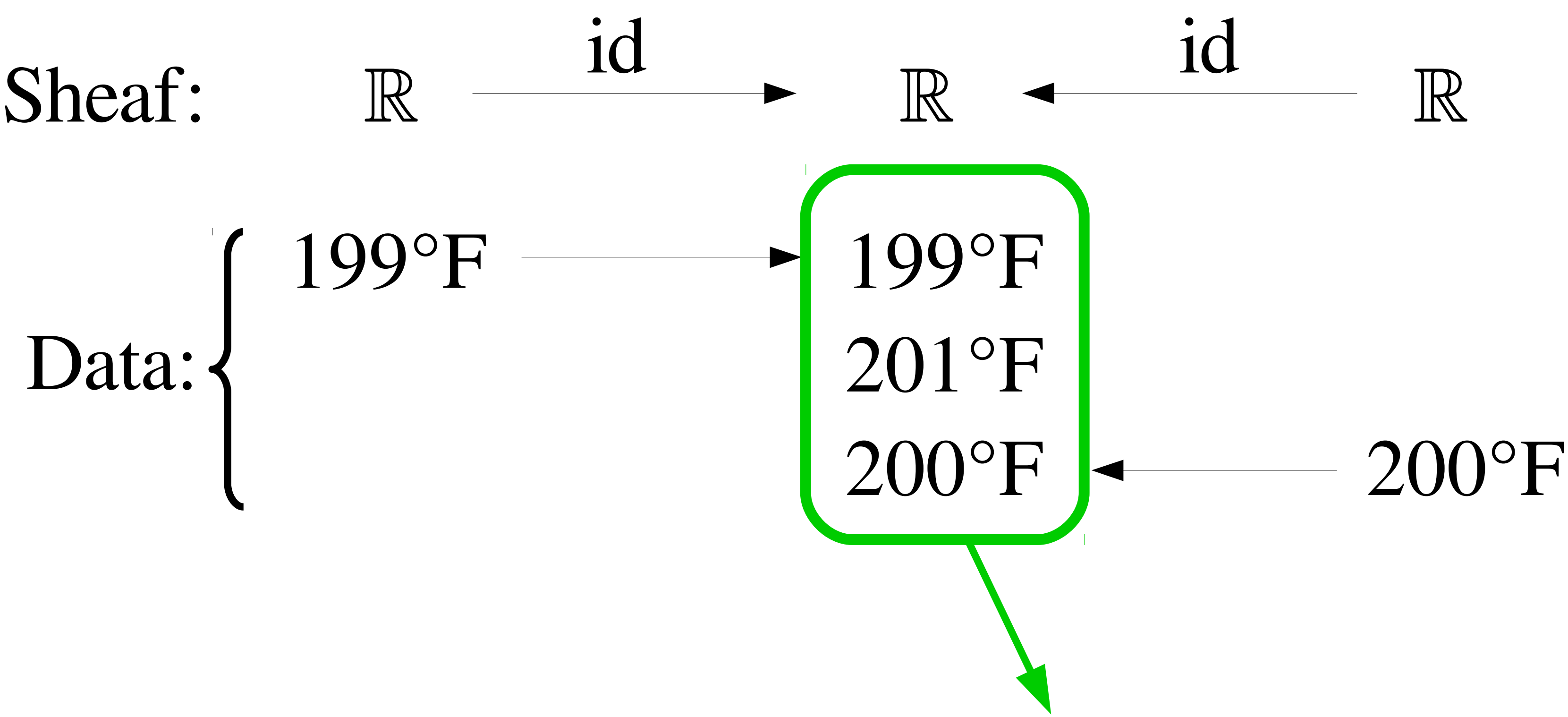
Relaxing matching requirements

- … any inconsistency cannot be tolerated…



Relaxing matching requirements

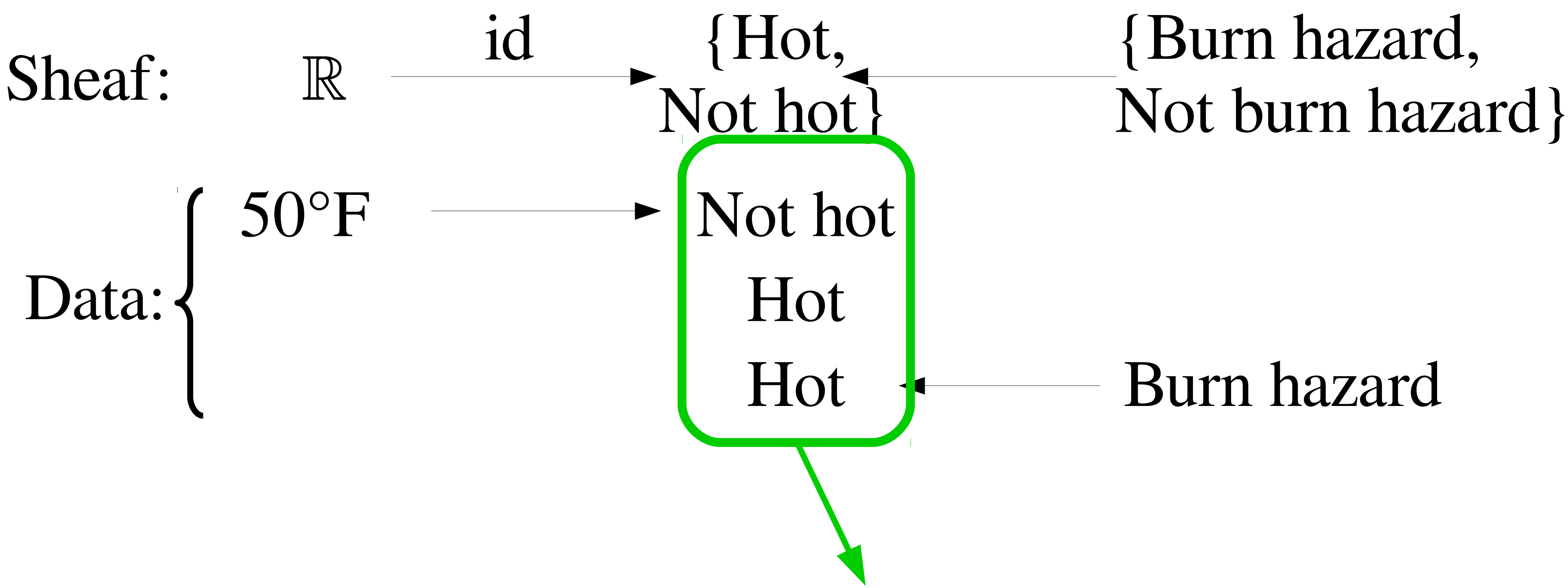
- … so relax the matching condition to handle errors



Variance({199,201,200}) < 10% → it's a *pseudosection*

Relaxing matching requirements

- … so it can do more than just check consistency



If any([Not hot, Hot, Hot]==Hot) →
Perhaps you should not touch!

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Consistency structures

Given a sheaf $\mathscr{S}$ on a simplicial complex X, one also needs a *consistency structure*:

- Assign to each non-vertex k-simplex a, a function

$$C_a : \mathscr{S}(a)^{2+k} \to \{0,1\}$$

- A *pseudosection* $p \in \oplus \mathscr{S}(a)$ satisfies

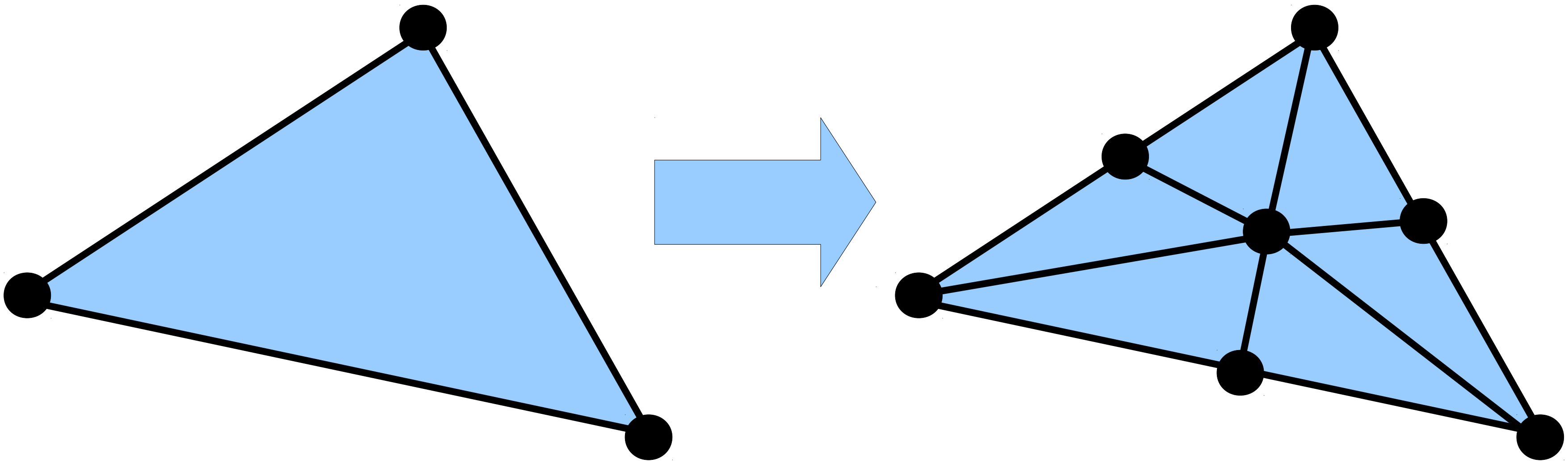
$$C_a(p(a), \mathscr{S}(v_0 \rightsquigarrow a)p(v_0), \ldots, \mathscr{S}(v_k \rightsquigarrow a)p(v_k)) = 1$$

everywhere it's defined, assuming $a = (v_0, \ldots, v_k)$.

- The consistency structure C_a returns 1 whenever the data at a are consistent

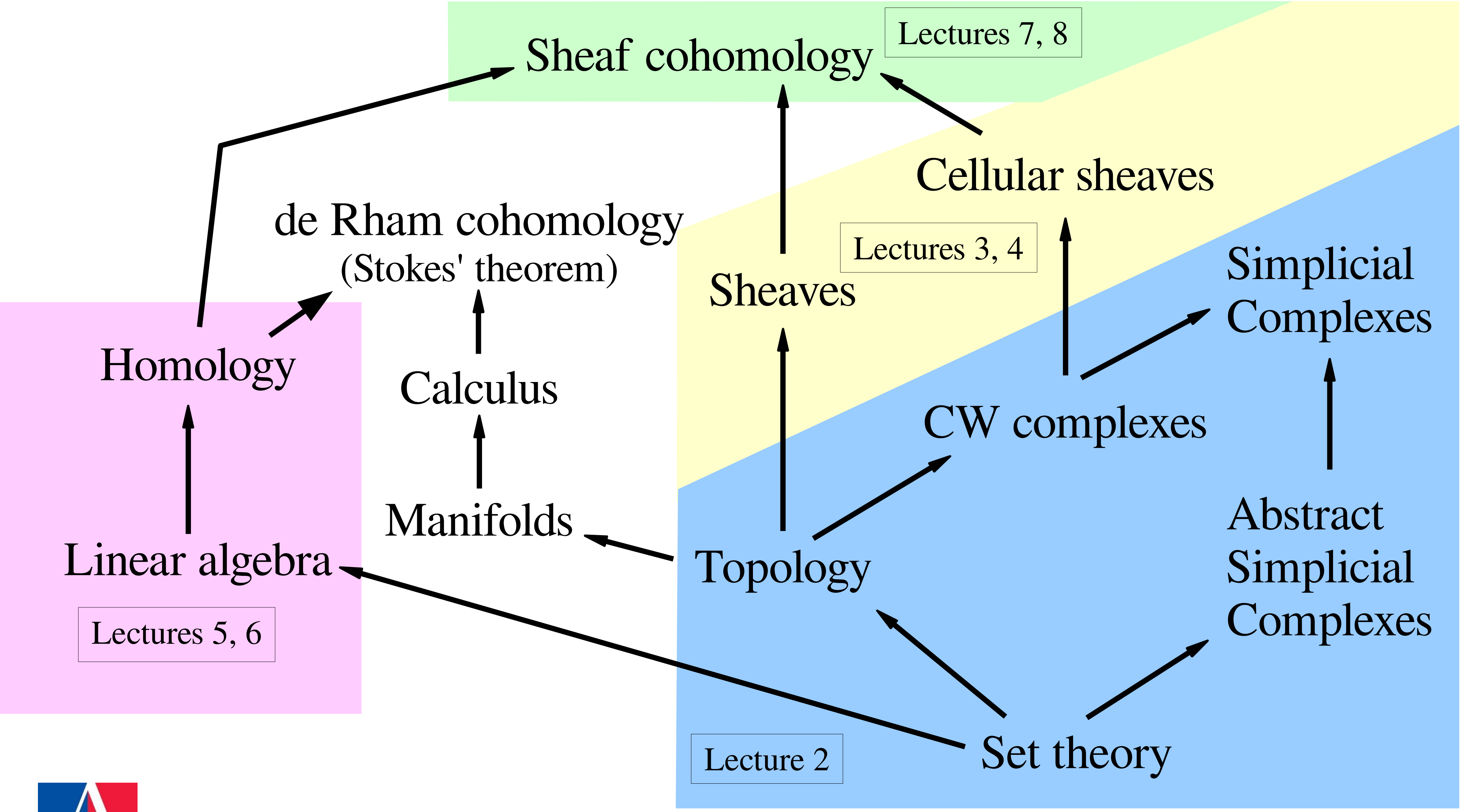
Pseudosections **are** sections…

- … just of a different sheaf
- <u>Theorem</u>: Pseudosections of a sheaf over an abstract simplicial complex X are sections of another sheaf over the barycentric subdivision of X



<u>Conclusion</u>: at least theoretically, it suffices to work with sheaves

Mathematical dependency tree



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Tutorial objectives

- What are sheaves?
  - The "local to global" viewpoint
- Encoding existing data into sheaves
  - "Sheafification"
- Data analytic capabilities enabled by sheaves
  - "Sections," "cohomology"
- Practice analyzing sheaves in software
  - "Persistence," "local homology"
- Interpret this analysis into the context of the data

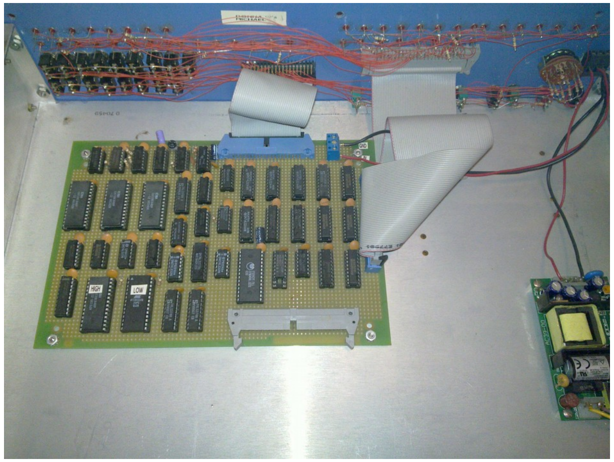
What's next?

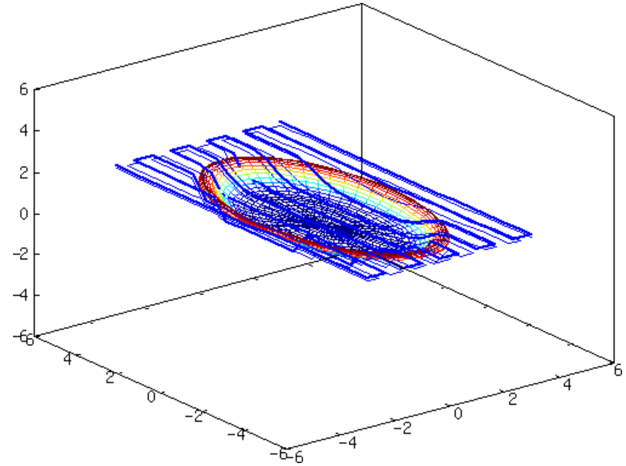
There are many open questions remaining… some will be addressed by DARPA SIMPLEX, but not all

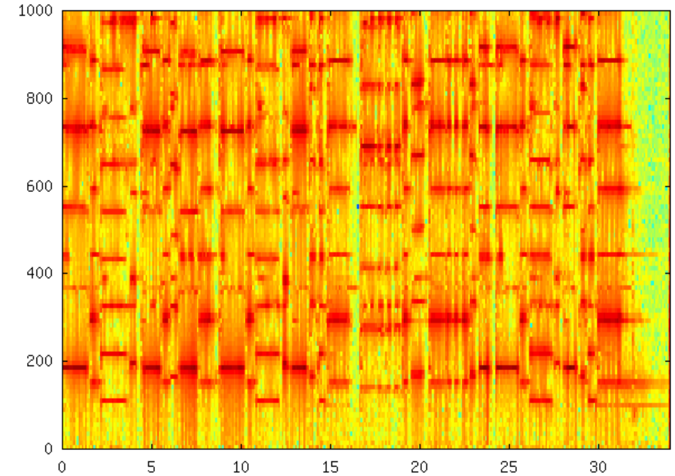
- Focus: heterogeneity, hypothesis generation

Wide open areas with little coverage in the literature:

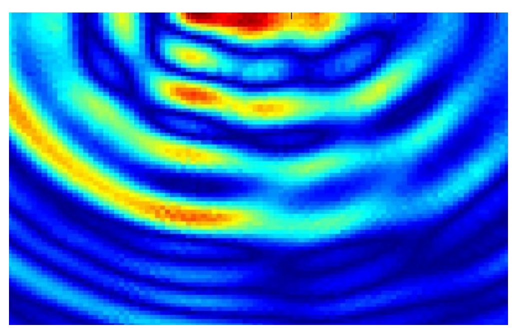
- Persistence for sheaves
- Duality relationships between sheaves and cosheaves
- Sheaf computations (cohomological or otherwise)
- Seriously addressing large or varied datasets







The biggest engineering problems are usually based on fun math problems



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Mathematician's view of the world

Applications are merely corollaries of great theorems

Differential equations Linear algebra Numerical analysis Dynamical systems Computational theory Logic	→ Physics → Data processing → Control and modeling → Computer hardware and software

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Applications lead to pure math

Physics	→	Differential equations
Data processing	→	Linear algebra Numerical analysis
Control and modeling	→	Dynamical systems
Computer hardware and software	→	Computational theory Logic

Further reading...

- Herbert Edelsbrunner and John Harer, "Persistent homology: A survey," *Surveys on Discrete and Computational Geometry. Twenty Years Later*, 257–282 (J. E. Goodman, J. Pach, and R. Pollack, eds.), Contemporary Mathematics 453, Amer. Math. Soc., Providence, Rhode Island, 2008.
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- Michael Robinson, "Multipath-dominant, pulsed doppler analysis of rotating blades," *IET Radar Sonar and Navigation*, Volume 7, Issue 3, March 2013, pp. 217-224.
- Michael Robinson and Robert Ghrist "Topological localization via signals of opportunity," *IEEE Trans. Sig. Proc*, Vol. 60, No. 5, May 2012.
- Shmuel Weinberger, "What is persistent homology?" *Notices of the Amer. Math. Soc.*, Vol. 58, No. 1, January 2011.

The End!

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http://www.drmichaelrobinson.net/