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**Problem 1.** Suppose  $A$  is a  $5 \times 7$  matrix. If rank of  $A = 4$ , then nullity of  $A =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

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**Problem 2.** Suppose  $A = PDP^{-1}$  where  $D$  is a diagonal matrix. Suppose also the  $d_{ii}$  are the diagonal entries of  $D$ . If  $P = [\vec{p}_1 \vec{p}_2 \vec{p}_3]$  and  $d_{11} = d_{33}$ , then  $\vec{p}_1 + \vec{p}_3$  is an eigenvector of  $A$

- A. True
- B. False

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**Problem 3.**

Suppose the orthogonal projection of  $\begin{bmatrix} -4 \\ 7 \\ 2 \end{bmatrix}$  onto  $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$  is  $(z_1, z_2, z_3)$ . Then  $z_1 =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

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**Problem 4.**

Let  $A = \begin{bmatrix} 8 & -24 & 32 \\ 0 & 2 & 8 \\ 0 & 0 & 8 \end{bmatrix}$ . Is  $A$  = diagonalizable?

- A. yes
- B. no
- C. none of the above

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**Problem 5.** The vector  $\vec{b}$  is in  $ColA$  if and only if  $A\vec{v} = \vec{b}$  has a solution

- A. True
- B. False

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**Problem 6.**

Let  $A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$ . Is  $A$  = diagonalizable?

- A. yes
- B. no
- C. none of the above

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**Problem 7.**

Calculate the determinant of  $\begin{bmatrix} -1.125 & -1 \\ 8 & 8 \end{bmatrix}$ .

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

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**Problem 8.** Suppose  $A$  is a  $3 \times 4$  matrix. Then  $\text{nul } A$  is a subspace of  $R^k$  where  $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

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**Problem 9.** Suppose  $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$  is a unit vector in the direction of  $\begin{bmatrix} 5 \\ 2 \\ 3.17214438511238 \end{bmatrix}$ . Then  $u_1 =$

- A. -0.8
- B. -0.6
- C. -0.4
- D. -0.2
- E. 0
- F. 0.2
- G. 0.4
- H. 0.6
- I. 0.8
- J. 1

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**Problem 10.** If  $\vec{x}_1$  and  $\vec{x}_2$  are solutions to  $A\vec{x} = \vec{b}$ , then  $-5\vec{x}_1 + 8\vec{x}_2$  is also a solution to  $A\vec{x} = \vec{b}$ .

- A. True
- B. False

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**Problem 11.**

Which of the following is an eigenvalue of  $\begin{bmatrix} 4 & 4 \\ 1 & 4 \end{bmatrix}$ .

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

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**Problem 12.**

$$\text{Let } A = \begin{bmatrix} 5.31034482758621 & 2.12413793103448 & -5.7448275862069 \\ 4.22413793103448 & 1.68965517241379 & -5.7448275862069 \\ 0 & 0 & -2.46206896551724 \end{bmatrix}$$

$$\text{and let } P = \begin{bmatrix} -2 & -4 & 7 \\ 5 & -7 & 7 \\ 0 & -8 & 3 \end{bmatrix}.$$

Suppose  $A = PDP^{-1}$ . Then if  $d_{ii}$  are the diagonal entries of  $D$ ,  $d_{11} =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

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**Problem 13.** Suppose  $A \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ . Then an eigenvalue of  $A$  is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

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**Problem 14.** Suppose  $A$  is a square matrix and  $A\vec{x} = \vec{0}$  has an infinite number of solutions, then given a vector  $\vec{b}$  of the appropriate dimension,  $A\vec{x} = \vec{b}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

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**Problem 15.**

Let  $A = \begin{bmatrix} 15 & -6 \\ 5 & -2 \end{bmatrix}$ .

Which of the following could be a basis for  $\text{null}(A)$ ?

- A.  $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- B.  $\left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$
- C.  $\left\{ \begin{bmatrix} 15 \\ 5 \end{bmatrix} \right\}$
- D.  $\left\{ \begin{bmatrix} 5 \\ -2 \end{bmatrix} \right\}$
- E.  $\left\{ \begin{bmatrix} 15 \\ 5 \end{bmatrix}, \begin{bmatrix} -6 \\ -2 \end{bmatrix} \right\}$
- F.  $\left\{ \begin{bmatrix} 15 \\ -6 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \end{bmatrix} \right\}$
- G.  $\mathbb{R}^2$
- H. none of the above