

1. (1 pt) local/Library/UI/LinearSystems/mformsHint.pg

Determine the following equivalent representations of the following system of equations:

$$3x + 8y = -7$$

$$-8x + 7y = -38$$

a. Find the augmented matrix of the system.

$$\begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$$

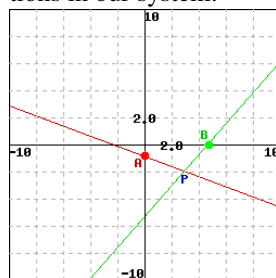
b. Find the matrix form of the system.

$$\begin{bmatrix} \_ & \_ \\ \_ & \_ \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \_ \\ \_ \end{bmatrix}$$

c. Find matrices that satisfy the following matrix equation.

$$x \begin{bmatrix} \_ \\ \_ \end{bmatrix} + y \begin{bmatrix} \_ \\ \_ \end{bmatrix} = \begin{bmatrix} \_ \\ \_ \end{bmatrix}$$

d. The graph below shows the lines determined by the two equations in our system:



Find the coordinates of

$P = (\_, \_)$

Find the coordinates of y-intercept of the red line.

$A = (0, \_)$

Find the coordinates of x-intercept of the green line.

$B = (\_, 0)$

**Hint:** (Instructor hint preview: show the student hint after 1 attempts. The current number of attempts is 0.)

For part d, note that you are given the equations of the red and green lines. If a point lies on a line, it must satisfy the equation for that line.

2. (1 pt) Library/Rochester/setLinearAlgebra9Dependence/urJa.9.10.pg

Express the vector  $v = \begin{bmatrix} 26 \\ 13 \end{bmatrix}$  as a linear combination of

$$x = \begin{bmatrix} 6 \\ 5 \end{bmatrix} \text{ and } y = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$$

$$v = \_ x + \_ y.$$

3. (1 pt) local/Library/UI/LinearSystems/linearCombHW4.pg

$$\text{Let } A = \begin{bmatrix} 1 & -1 & -3 \\ 4 & -2 & -10 \\ -2 & 1 & 5 \end{bmatrix} \text{ and } b = \begin{bmatrix} -1 \\ -11 \\ 5 \end{bmatrix}$$

? 1. Determine if  $b$  is a linear combination of  $a_1$ ,  $a_2$  and  $a_3$ , the columns of the matrix  $A$ .

If it is a linear combination, determine a non-trivial linear relation - (a non-trivial relation is three numbers which are not all three zero.) Otherwise, enter 0's for the coefficients.

$$\_ a_1 + \_ a_2 + \_ a_3 = b.$$

4. (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps.1-4/2.2.8.pg

$$\text{Let } \mathbf{a}_1 = \begin{bmatrix} -6 \\ -1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} -24 \\ -4 \end{bmatrix}.$$

Is  $\mathbf{b}$  in the span of  $\mathbf{a}_1$ ?

- A. No,  $\mathbf{b}$  is not in the span.
- B. Yes,  $\mathbf{b}$  is in the span.
- C. We cannot tell if  $\mathbf{b}$  is in the span.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.

$$\mathbf{b} = \_ \mathbf{a}_1$$

5. (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps.1-4/2.2.10.pg

$$\text{Let } \mathbf{a}_1 = \begin{bmatrix} -6 \\ 4 \\ 7 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} -7 \\ 2 \\ 9 \end{bmatrix}, \text{ and } \mathbf{b} = \begin{bmatrix} -31 \\ 18 \\ 37 \end{bmatrix}.$$

Is  $\mathbf{b}$  in the span of  $\mathbf{a}_1$  and  $\mathbf{a}_2$ ?

- A. Yes,  $\mathbf{b}$  is in the span.
- B. No,  $\mathbf{b}$  is not in the span.
- C. We cannot tell if  $\mathbf{b}$  is in the span.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.

$$\mathbf{b} = \_ \mathbf{a}_1 + \_ \mathbf{a}_2$$

6. (1 pt) local/Library/UI/LinearSystems/spanHW4.pg

$$\text{Let } A = \begin{bmatrix} 1 & -2 & 5 \\ 5 & -11 & 27 \\ 1 & -5 & 12 \end{bmatrix}, \text{ and } b = \begin{bmatrix} -4 \\ 2 \\ -4 \end{bmatrix}.$$

Denote the columns of  $A$  by  $a_1$ ,  $a_2$ ,  $a_3$ , and let  $W = \text{span}\{a_1, a_2, a_3\}$ .

? 1. Determine if  $b$  is in  $\{a_1, a_2, a_3\}$

? 2. Determine if  $b$  is in  $W$

How many vectors are in  $\{a_1, a_2, a_3\}$ ? (For infinitely many, enter -1) \_\_\_\_\_

How many vectors are in  $W$ ? (For infinitely many, enter -1) \_\_\_\_\_

**7. (1 pt) Library/TCNJ/TCNJ\_VectorEquations/problem4.pg**

Let  $u = \begin{bmatrix} -1 \\ 5 \\ -3 \end{bmatrix}$  and  $v = \begin{bmatrix} 1 \\ -7 \\ 4 \end{bmatrix}$

Find two vectors in  $\text{span}\{u, v\}$  that are not multiples of  $u$  or  $v$  and show the weights on  $u$  and  $v$  used to generate them.

\_\_\_\_  $u$  + \_\_\_\_  $v$  = \_\_\_\_\_

\_\_\_\_  $u$  + \_\_\_\_  $v$  = \_\_\_\_\_

**8. (1 pt) Library/TCNJ/TCNJ\_VectorEquations/problem6.pg**

Let  $\vec{u} = \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$  and  $\vec{v} = \begin{bmatrix} -10 \\ 1 \\ 0 \end{bmatrix}$ . Find a vector  $\vec{w}$  **not** in  $\text{span}\{\vec{u}, \vec{v}\}$ .

$\vec{w} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}$

**9. (1 pt) Library/TCNJ/TCNJ\_MatrixEquations/problem4.pg**

Let  $A = \begin{bmatrix} -1 & 1 & 5 \\ -2 & 3 & 3 \\ -2 & -3 & 1 \end{bmatrix}$  and  $x = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$ .

☐ 1. What does  $Ax$  mean?

**10. (1 pt) Library/Rochester/setLinearAlgebra9Dependence/ur\_1a.9.8.pg**

Let  $v_1 = \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$ ,  $v_2 = \begin{bmatrix} -2 \\ -6 \\ -1 \end{bmatrix}$ , and  $y = \begin{bmatrix} 2 \\ 10 \\ h \end{bmatrix}$ .

For what value of  $h$  is  $y$  in the plane spanned by  $v_1$  and  $v_2$ ?  
 $h =$  \_\_\_\_\_

**11. (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps.1-4/2.2.31.pg**

Let  $A = \begin{bmatrix} 3 & 3 \\ 5 & 4 \\ -3 & 8 \end{bmatrix}$ .

We want to determine if the system  $Ax = \mathbf{b}$  has a solution for every  $\mathbf{b} \in \mathbb{R}^3$ .

Select the best answer.

- A. There is a solution for every  $\mathbf{b} \in \mathbb{R}^3$  but we need to row reduce  $A$  to show this.
- B. There is a solution for every  $\mathbf{b} \in \mathbb{R}^3$  since  $2 < 3$
- C. There is a not solution for every  $\mathbf{b} \in \mathbb{R}^3$  but we need to row reduce  $A$  to show this.
- D. There is not a solution for every  $\mathbf{b} \in \mathbb{R}^3$  since  $2 < 3$ .

- E. We cannot tell if there is a solution for every  $\mathbf{b} \in \mathbb{R}^3$ .

**12. (1 pt) Library/TCNJ/TCNJ\_VectorEquations/problem5.pg**

Let  $H = \text{span}\{u, v\}$ . For each of the following sets of vectors determine whether  $H$  is a line or a plane.

☐ 1.  $u = \begin{bmatrix} -8 \\ -3 \\ -1 \end{bmatrix}$ ,  $v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ,

☐ 2.  $u = \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}$ ,  $v = \begin{bmatrix} -16 \\ -15 \\ -13 \end{bmatrix}$ ,

☐ 3.  $u = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$ ,  $v = \begin{bmatrix} -8 \\ 4 \\ 12 \end{bmatrix}$ ,

☐ 4.  $u = \begin{bmatrix} 5 \\ 1 \\ -2 \end{bmatrix}$ ,  $v = \begin{bmatrix} 13 \\ 2 \\ -2 \end{bmatrix}$ ,

**13. (1 pt) Library/TCNJ/TCNJ\_MatrixEquations/problem13.pg**

Do the following sets of vectors span  $\mathbb{R}^3$ ?

☐ 1.  $\begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ 6 \\ -7 \end{bmatrix}$ ,  $\begin{bmatrix} -7 \\ 14 \\ -16 \end{bmatrix}$ ,  $\begin{bmatrix} -11 \\ 22 \\ -26 \end{bmatrix}$

☐ 2.  $\begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ 2 \\ -4 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ 2 \\ -5 \end{bmatrix}$

☐ 3.  $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ ,  $\begin{bmatrix} -3 \\ -3 \\ 7 \end{bmatrix}$

☐ 4.  $\begin{bmatrix} 1 \\ -3 \\ 3 \end{bmatrix}$ ,  $\begin{bmatrix} -7 \\ 5 \\ 7 \end{bmatrix}$ ,  $\begin{bmatrix} -12 \\ 7 \\ 17 \end{bmatrix}$

**14. (1 pt) Library/TCNJ/TCNJ\_MatrixEquations/problem1.pg**

Show that the vectors

$\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$

do not span  $\mathbb{R}^3$  by giving a vector not in their span.

$\begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}$

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If a linear system has four equations and seven variables, then it must have infinitely many solutions.

- A. True
- B. False

Every linear system with free variables has infinitely many solutions.

- A. True
- B. False

Any linear system with more variables than equations cannot have a unique solution.

- A. True
- B. False

If a linear system has the same number of equations and variables, then it must have a unique solution.

- A. True
- B. False

A vector  $b$  is a linear combination of the columns of a matrix  $A$  if and only if the equation  $Ax = b$  has at least one solution.

- A. True
- B. False

If the columns of an  $m \times n$  matrix,  $A$  span  $\mathbb{R}^m$ , then the equation,  $Ax = b$  is consistent for each  $b$  in  $\mathbb{R}^m$ .

- A. True
- B. False

If  $A$  is an  $m \times n$  matrix and if the equation  $Ax = b$  is inconsistent for some  $b$  in  $\mathbb{R}^m$ , then  $A$  cannot have a pivot position in every row.

- A. True
- B. False

Any linear combination of vectors can always be written in the form  $Ax$  for a suitable matrix  $A$  and vector  $x$ .

- A. True
- B. False

If the equation  $Ax = b$  is inconsistent, then  $b$  is not in the set spanned by the columns of  $A$ .

- A. True
- B. False

If  $A$  is an  $m \times n$  matrix whose columns do not span  $\mathbb{R}^m$ , then the equation  $Ax = b$  is inconsistent for some  $b$  in  $\mathbb{R}^m$ .

- A. True
- B. False

The equation  $Ax = b$  is consistent if the augmented matrix  $[A \ b]$  has a pivot position in every row.

- A. True
- B. False

If the augmented matrix  $[A \ b]$  has a pivot position in every row, then the equation  $Ax = b$  is inconsistent.

- A. True
- B. False

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**16.** (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps.1-4-/2.2.56.pg

What conditions on a matrix  $A$  insures that  $A\mathbf{x} = \mathbf{b}$  has a solution for all  $\mathbf{b}$  in  $\mathbb{R}^n$ ?

Select the best statement. (The best condition should work with any positive integer  $n$ .)

- A. The equation will have a solution for all  $\mathbf{b}$  in  $\mathbb{R}^n$  as long as the columns of  $A$  do not include the zero column.
- B. The equation will have a solution for all  $\mathbf{b}$  in  $\mathbb{R}^n$  as long as the columns of  $A$  span  $\mathbb{R}^n$ .
- C. There is no easy test to determine if the equation will have a solution for all  $\mathbf{b}$  in  $\mathbb{R}^n$ .
- D. The equation will have a solution for all  $\mathbf{b}$  in  $\mathbb{R}^n$  as long as no column of  $A$  is a scalar multiple of another column.

- E. none of the above

**17.** (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps.1-4-/2.2.57.pg

Assume  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  spans  $\mathbb{R}^3$ .  
Select the best statement.

- A.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  spans  $\mathbb{R}^3$  unless  $\mathbf{u}_4$  is the zero vector.
- B.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  always spans  $\mathbb{R}^3$ .
- C. There is no easy way to determine if  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  spans  $\mathbb{R}^3$ .
- D.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  never spans  $\mathbb{R}^3$ .
- E.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  spans  $\mathbb{R}^3$  unless  $\mathbf{u}_4$  is a scalar multiple of another vector in the set.
- F. none of the above

**18.** (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps.1-4-/2.2.58.pg

Assume  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  does not span  $\mathbb{R}^3$ .  
Select the best statement.

- A.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  may, but does not have to, span  $\mathbb{R}^3$ .
- B.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  never spans  $\mathbb{R}^3$ .
- C.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  spans  $\mathbb{R}^3$  unless  $\mathbf{u}_4$  is a scalar multiple of another vector in the set.
- D.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  spans  $\mathbb{R}^3$  unless  $\mathbf{u}_4$  is the zero vector.
- E.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  always spans  $\mathbb{R}^3$ .
- F. none of the above

**19.** (1 pt) Library/TCNJ/TCNJ\_SolutionSetsLinearSystems-/problem8.pg

Suppose the solution set of a certain system of equations can be described as  $x_1 = 4 - 6t$ ,  $x_2 = 4 + 6t$ ,  $x_3 = 4t - 3$ ,  $x_4 = -6 - 4t$ , where  $t$  is a free variable. Use vectors to describe this solution set as a line in  $\mathbb{R}^4$ .

$$L(t) = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + t \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}.$$

**20.** (1 pt) Library/NAU/setLinearAlgebra/HomLinEq.pg  
Solve the equation

$$-6x - 2y + 9z = 0$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = s \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + t \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}$$

**21.** (1 pt) local/Library/UI/LinearSystems/ur\_la.1.19AxB.pg  
Solve the system

$$\begin{cases} 4x_1 - 3x_2 + 4x_3 + 4x_4 = 0 \\ -x_1 + x_2 + 3x_3 + 3x_4 = 0 \\ 3x_1 - 2x_2 + 7x_3 + 7x_4 = 0 \\ 3x_1 - 3x_2 - 9x_3 - 9x_4 = 0 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + s \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + t \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}.$$

Solve the system

$$\begin{cases} 4x_1 - 3x_2 + 4x_3 + 4x_4 = 3 \\ -x_1 + x_2 + 3x_3 + 3x_4 = 4 \\ 3x_1 - 2x_2 + 7x_3 + 7x_4 = 7 \\ 3x_1 - 3x_2 - 9x_3 - 9x_4 = -12 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} s + \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} t.$$

If the matrix  $A$  corresponds to the coefficient matrix for the above system of equations, then given any vector  $\vec{b}$ , the matrix equation  $A\vec{x} = \vec{b}$  will always have an infinite number of solutions.

- A. True
- B. False

**22.** (1 pt) local/Library/UI/LinearSystems/ur\_la.1.20vv3.pg  
Solve the system

$$\begin{cases} x_1 + 4x_2 + 2x_3 + 2x_5 - 2x_6 = 0 \\ -x_4 + 4x_5 - 4x_6 = 0 \\ x_1 + 4x_2 + 6x_5 - 4x_6 = 0 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + s \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + t \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix} + u \begin{bmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{bmatrix}.$$

Solve the system

$$\begin{cases} x_1 + 4x_2 + 2x_3 & + 2x_5 - 2x_6 = 1 \\ & -x_4 + 4x_5 - 4x_6 = 7 \\ x_1 + 4x_2 & + 6x_5 - 4x_6 = 5 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \\ \_ \\ \_ \end{bmatrix} + \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \\ \_ \\ \_ \end{bmatrix} s + \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \\ \_ \\ \_ \end{bmatrix} t + \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \\ \_ \\ \_ \end{bmatrix} u.$$

If the matrix  $A$  corresponds to the coefficient matrix for the above system of equations, then given any vector  $\vec{b}$ , the matrix equation  $A\vec{x} = \vec{b}$  will always has an infinite number of solutions.

- A. True
- B. False