Assignment HW7fall14 due 10/16/2014 at 11:59pm CDT

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1. **(1 pt) local/Library/UI/Fall14/HW7_4.pg**

Determine if the subset of $\mathbb{R}^2$ consisting of vectors of the form $\begin{bmatrix} a \\ b \end{bmatrix}$, where a and b are integers, is a subspace.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

The vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is included in the set, but the vector $(1/2) * \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$ is not included in the set.

Correct Answers:

- `A`
- `A`
- `B`
- `B`

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2. **(1 pt) local/Library/UI/Fall14/HW7_5.pg**

Determine if the subset of $\mathbb{R}^3$ consisting of vectors of the form $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, where $a \geq 0$, $b \geq 0$, and $c \geq 0$ is a subspace.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

The vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is included in the set, but the vector $(-1) * \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ is not included in the set.

Correct Answers:

- `A`
- `A`
- `B`
- `B`

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3. **(1 pt) local/Library/UI/Fall14/HW7_6.pg**

If A is an $n \times n$ matrix and $\mathbf{b} \neq 0$ in $\mathbb{R}^n$, then consider the set of solutions to $A\mathbf{x} = \mathbf{b}$.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

$A\mathbf{0} = \mathbf{0} \neq \mathbf{b}$, so the zero vector is not in the set and it is not a subspace.

Correct Answers:

- `B`
- `B`
- `B`
- `B`

\end{array}\right.\)

4. **(1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/4.1.77.pg**

The null space for the matrix $\begin{bmatrix} 1 & 7 & -2 & 14 & 0 \\ 3 & 0 & 1 & -2 & 3 \\ 6 & 1 & -1 & 0 & 4 \end{bmatrix}$

is spanA,B where A = $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$ B = $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION:
We can use a CAS to get

$$\text{null}\left(\begin{bmatrix} 1 & 7 & -2 & 14 & 0 \\ 3 & 0 & 1 & -2 & 3 \\ 6 & 1 & -1 & 0 & 4 \end{bmatrix}\right) = \text{span}\left(\begin{bmatrix} 0.428571428571429 \\ -1.85714285714286 \\ 0.714285714285714 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -0.767857142857143 \\ 0.0892857142857143 \\ -0.696428571428571 \\ 0 \\ 1 \end{bmatrix}\right)$$

Correct Answers:

- 0.428571428571429
- -1.85714285714286
- 0.714285714285714
- 1
- 0
- -0.767857142857143
- 0.0892857142857143
- -0.696428571428571
- 0
- 1

5. **(1 pt) Library/Rochester/setLinearAlgebra10Bases/ur_la_10_26.pg**

Find a basis of the subspace of $\mathbb{R}^4$ defined by the equation $-7x_1 - 3x_2 - 8x_3 - 8x_4 = 0$.

$\begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}, \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}, \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}.$

Correct Answers:

- \(\displaystyle\left.\begin{array}{c}
\mbox{-3} \cr
\mbox{7} \cr
\mbox{0} \cr
\mbox{0} \cr
\end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
\mbox{-8} \cr
\mbox{0} \cr
\mbox{7} \cr
\mbox{0} \cr
\end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
\mbox{-8} \cr
\mbox{0} \cr
\mbox{0} \cr
\mbox{7} \cr

6. **(1 pt) local/Library/UI/6a.pg**

The null space for the matrix $\begin{bmatrix} 2 & 0 & 5 \\ -1 & 6 & 2 \\ 4 & 4 & -1 \\ 5 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix}$

is $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION:

$$\text{null}\left(\begin{bmatrix} 2 & 0 & 5 \\ -1 & 6 & 2 \\ 4 & 4 & -1 \\ 5 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix}\right) = \left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$$

Correct Answers:

- 0
- 0
- 0

7. **(1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/4.2.32a.pg**

Find a basis for the null space of matrix A.

A = $\begin{bmatrix} 1 & 0 & -4 & 5 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$

Basis = $\begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \\ \_ \end{bmatrix} \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}$

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION:
Row-reduce the matrix which has the given vectors as columns.

A is already row-reduced, thus $A\mathbf{x} = \mathbf{0}$ has solutions of the form

$$\mathbf{x} = s_1 \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s_2 \begin{bmatrix} 14 \\ 3 \\ 0 \\ -3 \\ 1 \end{bmatrix}$$

so that a basis for the subspace is

$$\left\{ \begin{bmatrix} 4 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 14 \\ 3 \\ 0 \\ -3 \\ 1 \end{bmatrix} \right\}$$

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{4} \cr
  \mbox{0} \cr
  \mbox{1} \cr
  \mbox{0} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c}
  \mbox{14} \cr
  \mbox{3} \cr
  \mbox{0} \cr
  \mbox{-3} \cr
  \mbox{1} \cr
  \end{array}\right.\)
  ```

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8. **(1 pt) local/Library/Rochester/setLinearAlgebra3Matrices-/ur_la_3_14.pg**

Find the ranks of the following matrices.

$$rank \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 0 \end{bmatrix} = \_\_$$

$$rank \begin{bmatrix} -1 & 0 & -4 \\ 1 & 6 & 0 \\ -9 & 0 & 0 \end{bmatrix} = \_\_$$

$$rank \begin{bmatrix} 6 & 1 & -6 \\ 0 & 9 & 0 \\ -4 & 0 & 4 \end{bmatrix} = \_\_$$

Correct Answers:

- 3
- 3
- 2

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9. **(1 pt) local/Library/UI/Fall14/HW7_11.pg**

Find all values of x for which $\text{rank}(A) = 2$.

$$A = \begin{bmatrix} 2 & 1 & 0 & 7 \\ -2 & 2 & x & -7 \\ 3 & 7 & 4 & 28 \end{bmatrix}$$

$x =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION:

Row reduce A to get:

$$\begin{bmatrix} 2 & 1 & 0 & 7 \\ -2 & 2 & x & -7 \\ 3 & 7 & 4 & 28 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 0 & 7 \\ 0 & 4 & x & 7 \\ 0 & 12 & 12 & 21 \end{bmatrix}$$

Since two pivots are needed, $x = 4$

Correct Answers:

- I

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10. **(1 pt) local/Library/UI/Fall14/HW7_12.pg**

Suppose that A is a 8×6 matrix which has a null space of dimension 6. The rank of A=

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

Using the Rank-Nullity theorem, if the dimensions of A is n x m, rank(A) = m - nullity(A) = 6 - 6 = 0

Correct Answers:

- E

Suppose A is a 5×4 matrix. If rank of A = 1, then nullity of A =

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- H
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The vector $\vec{b}$ is NOT in $ColA$ if and only if $A\vec{v} = \vec{b}$ does NOT have a solution

- A. True
- B. False

Correct Answers:

- A

The vector $\vec{b}$ is in $ColA$ if and only if $A\vec{v} = \vec{b}$ has a solution

- A. True
- B. False

Correct Answers:

- A

The vector $\vec{v}$ is in $NulA$ if and only if $A\vec{v} = \vec{0}$

- A. True
- B. False

Correct Answers:

- A

If the equation $A\vec{x} = \vec{b_1}$ has at least one solution and if the equation $A\vec{x} = \vec{b_2}$ has at least one solution, then the equation $A\vec{x} = -1\vec{b_1} - 3\vec{b_2}$ also has at least one solution.

- A. True
- B. False

Correct Answers:

- A

If $\vec{x_1}$ and $\vec{x_2}$ are solutions to $A\vec{x} = \vec{0}$, then $8\vec{x_1} - 1\vec{x_2}$ is also a solution to $A\vec{x} = \vec{0}$.

- A. True
- B. False

Correct Answers:

- A

If $\vec{x_1}$ and $\vec{x_2}$ are solutions to $A\vec{x} = \vec{b}$, then $2\vec{x_1} - 9\vec{x_2}$ is also a solution to $A\vec{x} = \vec{b}$.

- A. True
- B. False

Correct Answers:

- B

Suppose A is a 4×2 matrix. Then *nul A* is a subspace of R^k where $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- G

Suppose A is a 2×6 matrix. Then *col A* is a subspace of R^k where $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- G

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20. **(1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/problem5.pg**

Let W_1 be the set: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

Determine if W_1 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_1 is a basis.
- B. W_1 is not a basis because it does not span $\mathbb{R}^3$.
- C. W_1 is not a basis because it is linearly dependent.

Let W_2 be the set: $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

Determine if W_2 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_2 is not a basis because it does not span $\mathbb{R}^3$.
- B. W_2 is a basis.

- C. W_2 is not a basis because it is linearly dependent.

Correct Answers:

- `A`
- `AC`

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21. **(1 pt) local/Library/UI/Fall14/HW7_25.pg**

Indicate whether the following statement is true or false?
If $S = \text{span} u_1, u_2, u_3$, then $dim(S) = 3$.

- A. True
- B. False

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION:
FALSE. For example, suppose
$S = \text{span}\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$,
then $dim(S) < 3$

Correct Answers:

- `B`

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22. **(1 pt) local/Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-/3.pg**

Check the true statements below:

- A. The columns of an invertible $n \times n$ matrix form a basis for $\mathbb{R}^n$.
- B. If $H = Span\{b_1, ..., b_p\}$, then $\{b_1, ..., b_p\}$ is a basis for H.
- C. If B is an echelon form of a matrix A, then the pivot columns of B form a basis for $ColA$.
- D. The column space of a matrix A is the set of solutions of $Ax = b$.
- E. A basis is a spanning set that is as large as possible.

Correct Answers:

- `A`

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23. **(1 pt) local/Library/UI/4.3.1a.pg**

Find bases for the column space and the null space of matrix A. You should verify that the Rank-Nullity Theorem holds. An equivalent echelon form of matrix A is given to make your work easier.

A = $\begin{bmatrix} 1 & 2 & 12 \\ 3 & 3 & 21 \\ 3 & 8 & 46 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

Basis for the column space of A = $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

Basis for the null space of A = $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION:
A basis for the column space, determined from the pivot columns 1 and 2, is
$\left\{ \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix} \right\}$

Solve $A\mathbf{x} = \mathbf{0}$, to obtain $\mathbf{x} = s \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix}$, and so the nullspace basis is $\left\{ \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} \right\}$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{1} \cr
  \mbox{3} \cr
  \mbox{3} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{2} \cr
  \mbox{3} \cr
  \mbox{8} \cr
  \end{array}\right.\)
  ```
- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{2} \cr
  \mbox{5} \cr
  \mbox{1} \cr
  \end{array}\right.\)
  ```

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24. **(1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/4.1.22.pg**

Find the null space for $A = \begin{bmatrix} 3 & 2 \\ 1 & -9 \end{bmatrix}$.

What is null(A)?

- A. span$\left\{ \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\}$
- B. span$\left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$
- C. $\mathbb{R}^2$
- D. span$\left\{ \begin{bmatrix} -2 \\ 3 \end{bmatrix} \right\}$
- E. $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- F. span$\left\{ \begin{bmatrix} -1 \\ 3 \end{bmatrix} \right\}$
- G. span$\left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$
- H. none of the above

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Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION
A row reduces to the identity matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\begin{bmatrix}0\\0\end{bmatrix}$,

and thus, $\text{null}(A) = \begin{bmatrix}0\\0\end{bmatrix}$.

Correct Answers:

- E

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25. **(1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/4.1.30.pg**

Find the null space for $A = \begin{bmatrix}2 & -3 & 7\\6 & 3 & -15\\3 & 5 & -18\end{bmatrix}$.

What is $\text{null}(A)$?

- A. $\text{span}\left\{\begin{bmatrix}2\\-3\\7\end{bmatrix}, \begin{bmatrix}6\\3\\-15\end{bmatrix}\right\}$
- B. $\mathbb{R}^3$
- C. $\text{span}\left\{\begin{bmatrix}2\\-3\\7\end{bmatrix}\right\}$
- D. $\text{span}\left\{\begin{bmatrix}1\\3\\1\end{bmatrix}\right\}$
- E. $\text{span}\left\{\begin{bmatrix}2\\6\\3\end{bmatrix}\right\}$
- F. $\text{span}\left\{\begin{bmatrix}0\\0\\0\end{bmatrix}\right\}$
- G. none of the above

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Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

A is row reduces to $\begin{bmatrix}1 & 0 & 1\\0 & 1 & 3\\0 & 0 & 0\end{bmatrix}$. The basis of the null space has one element for each column without a leading one in the row reduced matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has a one dimentional null space,

and $\text{null}(A)$ is the subspace generated by $\begin{bmatrix}1\\3\\1\end{bmatrix}$.

Correct Answers:

- D

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26. **(1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps_1-4-/4.1.28.pg**

Find the null space for $A = \begin{bmatrix}2 & 6\\7 & 21\\4 & 12\end{bmatrix}$.

What is $\text{null}(A)$?

- A. $\mathbb{R}^3$
- B. $\text{span}\left\{\begin{bmatrix}0\\0\\0\end{bmatrix}\right\}$
- C. $\mathbb{R}^2$
- D. $\text{span}\left\{\begin{bmatrix}12\\4\end{bmatrix}\right\}$
- E. $\text{span}\left\{\begin{bmatrix}-3\\1\end{bmatrix}\right\}$
- F. $\text{span}\left\{\begin{bmatrix}2\\7\\4\end{bmatrix}\right\}$
- G. $\text{span}\left\{\begin{bmatrix}0\\0\end{bmatrix}\right\}$
- H. none of the above

.

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION

A is row reduces to $\begin{bmatrix}2 & 6\\0 & 0\\0 & 0\end{bmatrix}$. The basis of the null space has one element for each column without a leading one in the row reduced matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has a one dimentional null space,

and $\text{null}(A)$ is the subspace generated by $\begin{bmatrix}-6\\2\end{bmatrix}$.

Correct Answers:

- E

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27. **(1 pt) local/Library/UI/4.3.3.pg**

Find bases for the column space and the null space of matrix A. You should verify that the Rank-Nullity Theorem holds. An equivalent echelon form of matrix A is given to make your work easier.

$$A = \begin{bmatrix}1 & 0 & -5 & -4\\-2 & 1 & 15 & 7\\0 & 1 & 5 & -1\end{bmatrix} \begin{bmatrix}1 & 0 & -5 & -4\\0 & 1 & 5 & -1\\0 & 0 & 0 & 0\end{bmatrix}$$

Basis for the column space of A = $\begin{bmatrix}\_\_\\\_\_\\\_\_\end{bmatrix} \begin{bmatrix}\_\_\\\_\_\\\_\_\end{bmatrix}$

Basis for the null space of A = $\begin{bmatrix}\_\_\\\_\_\\\_\_\\\_\_\end{bmatrix} \begin{bmatrix}\_\_\\\_\_\\\_\_\\\_\_\end{bmatrix}$

Solution: *(Instructor solution preview: show the student solution after due date.)*

SOLUTION:

A basis for the column space, determined from the pivot columns 1 and 2, is
$\left\{ \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

Solve $A\mathbf{x} = \mathbf{0}$, to obtain $\mathbf{x} = s_1 \begin{bmatrix} +5 \\ -5 \\ 1 \\ 0 \end{bmatrix} + s_2 \begin{bmatrix} +4 \\ +1 \\ 0 \\ 1 \end{bmatrix}$, and so the nullspace basis is $\left\{ \begin{bmatrix} +5 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} +4 \\ +1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

Correct Answers:

- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{1} \cr
  \mbox{-2} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{0} \cr
  \mbox{1} \cr
  \mbox{1} \cr
  \end{array}\right.\)
  ```
- ```
  \(\displaystyle\left.\begin{array}{c}
  \mbox{5} \cr
  \mbox{-5} \cr
  \mbox{1} \cr
  \mbox{0} \cr
  \end{array}\right.\) ,\(\displaystyle\left.\begin{array}{c
  \mbox{4} \cr
  \mbox{1} \cr
  \mbox{0} \cr
  \mbox{1} \cr
  \end{array}\right.\)
  ```