
Problem 1. Suppose A is a 5×7 matrix. If $\text{rank of } A = 4$, then nullity of $A =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 2. Suppose $A = PDP^{-1}$ where D is a diagonal matrix. Suppose also the d_{ii} are the diagonal entries of D . If $P = [\vec{p}_1 \ \vec{p}_2 \ \vec{p}_3]$ and $d_{11} = d_{33}$, then $\vec{p}_1 + \vec{p}_3$ is an eigenvector of A

- A. True
- B. False

Problem 3.

Suppose the orthogonal projection of $\begin{bmatrix} -4 \\ 7 \\ 2 \end{bmatrix}$ onto $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ is (z_1, z_2, z_3) . Then $z_1 =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 4.

Let $A = \begin{bmatrix} 8 & -24 & 32 \\ 0 & 2 & 8 \\ 0 & 0 & 8 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Problem 5. The vector \vec{b} is in $ColA$ if and only if $A\vec{v} = \vec{b}$ has a solution

- A. True
- B. False

Problem 6.

Let $A = \begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Problem 7.

Calculate the determinant of $\begin{bmatrix} -1.125 & -1 \\ 8 & 8 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

Problem 8. Suppose A is a 3×4 matrix. Then $\text{nul } A$ is a subspace of R^k where $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 9. Suppose $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ is a unit vector in the direction of $\begin{bmatrix} 5 \\ 2 \\ 3.17214438511238 \end{bmatrix}$. Then $u_1 =$

- A. -0.8
- B. -0.6
- C. -0.4
- D. -0.2
- E. 0
- F. 0.2
- G. 0.4
- H. 0.6
- I. 0.8
- J. 1

Problem 10. If \vec{x}_1 and \vec{x}_2 are solutions to $A\vec{x} = \vec{b}$, then $-5\vec{x}_1 + 8\vec{x}_2$ is also a solution to $A\vec{x} = \vec{b}$.

- A. True
- B. False

Problem 11.

Which of the following is an eigenvalue of $\begin{bmatrix} 4 & 4 \\ 1 & 4 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 12.

$$\text{Let } A = \begin{bmatrix} 5.31034482758621 & 2.12413793103448 & -5.7448275862069 \\ 4.22413793103448 & 1.68965517241379 & -5.7448275862069 \\ 0 & 0 & -2.46206896551724 \end{bmatrix}$$

$$\text{and let } P = \begin{bmatrix} -2 & -4 & 7 \\ 5 & -7 & 7 \\ 0 & -8 & 3 \end{bmatrix}.$$

Suppose $A = PDP^{-1}$. Then if d_{ii} are the diagonal entries of D , $d_{11} =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

Problem 13. Suppose $A \begin{bmatrix} -2 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Then an eigenvalue of A is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 14. Suppose A is a square matrix and $A\vec{x} = \vec{0}$ has an infinite number of solutions, then given a vector \vec{b} of the appropriate dimension, $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Problem 15.

Let $A = \begin{bmatrix} 15 & -6 \\ 5 & -2 \end{bmatrix}$.

Which of the following could be a basis for $\text{null}(A)$?

- A. $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- B. $\left\{ \begin{bmatrix} 2 \\ 5 \end{bmatrix} \right\}$
- C. $\left\{ \begin{bmatrix} 15 \\ 5 \end{bmatrix} \right\}$
- D. $\left\{ \begin{bmatrix} 5 \\ -2 \end{bmatrix} \right\}$
- E. $\left\{ \begin{bmatrix} 15 \\ 5 \end{bmatrix}, \begin{bmatrix} -6 \\ -2 \end{bmatrix} \right\}$
- F. $\left\{ \begin{bmatrix} 15 \\ -6 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \end{bmatrix} \right\}$
- G. \mathbb{R}^2
- H. none of the above