

[18] 1.) Find the characteristic equation and diagonalize  $A = \begin{bmatrix} -2 & 6 \\ 6 & -18 \end{bmatrix}$

Characteristic equation of  $A =$  \_\_\_\_\_.

$P =$  \_\_\_\_\_

$D =$  \_\_\_\_\_

$P^{-1} =$  \_\_\_\_\_.

[16] 2.) Use Gram-Schmidt to find the  $QR$  factorization of  $M = \begin{bmatrix} 6 & 3 \\ 6 & 0 \\ 3 & 3 \end{bmatrix}$ .

$Q =$  \_\_\_\_\_

$R =$  \_\_\_\_\_

Part 2: Multiple Choice (multiple choice problems are worth 6 points each).

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**Problem 1.** The eigenspace corresponding to a particular eigenvalue of  $A$  contains an infinite number of vectors.

- A. True
  - B. False
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**Problem 2.** Let  $A = \begin{bmatrix} -1 & -7 & -6 \\ 0 & 7 & -8 \\ 0 & 0 & -1 \end{bmatrix}$ . Is  $A$  diagonalizable?

- A. yes
  - B. no
  - C. none of the above
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**Problem 3.** Suppose  $A \begin{bmatrix} 4 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} -16 \\ 8 \\ 4 \end{bmatrix}$ . Then an eigenvalue of  $A$  is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

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**Problem 4.** 0 is an eigenvalue of  $A$  if and only if  $\det(A) = 0$

- A. True
- B. False

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**Problem 5.** If the characteristic polynomial of  $A = (\lambda + 1)^9(\lambda - 5)^2(\lambda + 2)^3$ , then the algebraic multiplicity of  $\lambda = 5$  is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

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**Problem 6.** Suppose  $A = PDP^{-1}$  where  $D$  is a diagonal matrix. Suppose also the  $d_{ii}$  are the diagonal entries of  $D$ . If  $P = [\vec{p}_1 \ \vec{p}_2 \ \vec{p}_3]$  and  $d_{11} = d_{22}$ , then  $4\vec{p}_1 + 3\vec{p}_2$  is an eigenvector of  $A$

- A. True
- B. False

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**Problem 7.** Let  $A = \begin{bmatrix} 3 & -1 & -6 \\ 0 & 5 & 12 \\ 0 & 0 & 3 \end{bmatrix}$ . Is  $A$  diagonalizable?

- A. yes
- B. no
- C. none of the above

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**Problem 8.**

If the characteristic polynomial of  $A = (\lambda - 8)^3(\lambda + 8)(\lambda + 1)^3$ , then the geometric multiplicity of  $\lambda = -8$  is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

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**Problem 9.** If  $x$  is not in a subspace  $W$ , then  $x - \text{proj}_W(x)$  is not zero.

- A. True
- B. False

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**Problem 10.** If  $A$  is symmetric, then  $A$  is diagonalizable.

- A. True
- B. False

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**Problem 11.** Suppose the orthogonal projection of  $\begin{bmatrix} -147 \\ 3 \\ 9 \end{bmatrix}$  onto  $\begin{bmatrix} 1 \\ -5 \\ 4 \end{bmatrix}$  is  $(z_1, z_2, z_3)$ . Then  $z_1 =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above