

Problem 1. Suppose A is a 9×8 matrix. If rank of $A = 5$, then nullity of $A =$

- A. -4
 - B. -3
 - C. -2
 - D. -1
 - E. 0
 - F. 1
 - G. 2
 - H. 3
 - I. 4
 - J. none of the above
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Problem 2. Suppose A is a square matrix and $A\vec{x} = \vec{0}$ has an infinite number of solutions, then given a vector \vec{b} of the appropriate dimension, $A\vec{x} = \vec{b}$ has

- A. No solution
 - B. Unique solution
 - C. Infinitely many solutions
 - D. at most one solution
 - E. either no solution or an infinite number of solutions
 - F. either a unique solution or an infinite number of solutions
 - G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
 - H. none of the above
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Problem 3. A vector b is a linear combination of the columns of a matrix A if and only if the equation $Ax = b$ has at least one solution.

- A. True
- B. False

Problem 4.

Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ does not span \mathbb{R}^3 .

Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans \mathbb{R}^3 .
 - B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is the zero vector.
 - C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is a scalar multiple of another vector in the set.
 - D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is in $\text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
 - E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans \mathbb{R}^3 .
 - F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ may, but does not have to, span \mathbb{R}^3 .
 - G. none of the above
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Problem 5. Given that $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 8 & -3 \\ 18 & -7 \end{bmatrix}$, determine the corresponding eigenvalue.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. None of those above

Problem 6. Suppose A is a 4×2 matrix. Then $\text{col } A$ is a subspace of R^k where $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 7. Let A be a matrix with linearly independent columns.
Select the best statement.

- A. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} precisely when it has more rows than columns.
- B. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} precisely when it is a square matrix.
- C. The equation $A\mathbf{x} = \mathbf{b}$ never has a solution for all \mathbf{b} .
- D. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} precisely when it has more columns than rows.
- E. There is insufficient information to determine if $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} .
- F. The equation $A\mathbf{x} = \mathbf{b}$ always has a solution for all \mathbf{b} .
- G. none of the above

Problem 8. If $A = \begin{bmatrix} -1 & 4 \\ -7 & 5 \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, and $AB = I$, the identity matrix, then $b_{11} =$

- A. $\frac{1}{23}$
- B. $\frac{4}{23}$
- C. $\frac{5}{23}$
- D. $\frac{7}{23}$
- E. $-\frac{1}{23}$
- F. $-\frac{4}{23}$
- G. $-\frac{5}{23}$
- H. $-\frac{7}{23}$
- I. 7
- J. None of those above

Problem 9. $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$

The determinant of the above matrix is

- A. -362880
- B. -40320
- C. -540
- D. -1
- E. 0
- F. 1
- G. 540
- H. 40320
- I. 362880
- J. None of the above

Problem 10. If \vec{x}_1 and \vec{x}_2 are solutions to $A\vec{x} = \vec{b}$, then $8\vec{x}_1 - 3\vec{x}_2$ is also a solution to $A\vec{x} = \vec{b}$.

- A. True
- B. False

Problem 11. The determinant of a square matrix A is nonzero if and only if the equation $Ax = 0$ has a unique solution.

- A. True
- B. False

Problem 12. Find the null space for $A = \begin{bmatrix} 1 & -4 \\ -1 & 4 \\ 4 & -16 \end{bmatrix}$.

What is $\text{null}(A)$?

- A. $\text{span}\left\{\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}\right\}$
- B. \mathbb{R}^2
- C. \mathbb{R}^3
- D. $\text{span}\left\{\begin{bmatrix} 4 \\ 1 \end{bmatrix}\right\}$
- E. $\text{span}\left\{\begin{bmatrix} -16 \\ 4 \end{bmatrix}\right\}$
- F. $\text{span}\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$
- G. $\text{span}\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$
- H. none of the above

Problem 13. Find all values of x for which $\text{rank}(A) = 2$. where $A = \begin{bmatrix} 1 & 2 & 0 & 7 \\ -2 & -2 & x & -5 \\ -3 & -8 & 2 & -30 \end{bmatrix}$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above