

[18] 1.) Find the characteristic equation and diagonalize $A = \begin{bmatrix} 4 & 8 \\ 8 & 16 \end{bmatrix}$

Characteristic equation of $A =$ _____.

$P =$ _____

$D =$ _____

$P^{-1} =$ _____.

[16] 2.) Use Gram-Schmidt to find the QR factorization of $M = \begin{bmatrix} 1 & 6 \\ 2 & 6 \\ 2 & 0 \end{bmatrix}$.

$Q =$ _____

$R =$ _____

Part 2: Multiple Choice (multiple choice problems are worth 6 points each).

Problem 1. There are an infinite number of eigenvectors that correspond to a particular eigenvalue of A

- A. True
 - B. False
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Problem 2. The vector $\mathbf{0}$ is an eigenvector of A if and only if $\det(A) = 0$.

- A. True
 - B. False
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Problem 3. If the characteristic polynomial of $A = (\lambda - 7)^1(\lambda + 7)^2(\lambda + 8)^6$, then the algebraic multiplicity of $\lambda = -7$ is

- A. 0
 - B. 1
 - C. 2
 - D. 3
 - E. 0 or 1
 - F. 0 or 2
 - G. 1 or 2
 - H. 0, 1, or 2
 - I. 0, 1, 2, or 3
 - J. none of the above
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Problem 4. Let $A = \begin{bmatrix} 4 & 5 & 10 \\ 0 & 2 & 2 \\ 0 & 0 & 4 \end{bmatrix}$. Is A diagonalizable?

- A. yes
- B. no
- C. none of the above

Problem 5. If A is diagonalizable, then A is symmetric.

- A. True
- B. False

Problem 6. Suppose $A = PDP^{-1}$ where D is a diagonal matrix. Suppose also the d_{ii} are the diagonal entries of D . If $P = [\vec{p}_1 \ \vec{p}_2 \ \vec{p}_3]$ and $d_{11} = d_{22}$, then $2\vec{p}_1 + 5\vec{p}_2$ is an eigenvector of A

- A. True
- B. False

Problem 7. If the characteristic polynomial of $A = (\lambda - 7)^9(\lambda - 3)^2(\lambda + 2)^4$, then the geometric multiplicity of $\lambda = 3$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

Problem 8. If x is in a subspace W , then $x - \text{proj}_W(x)$ is not zero.

- A. True
- B. False

Problem 9. Let $A = \begin{bmatrix} -4 & 5 & 10 \\ 0 & -3 & 2 \\ 0 & 0 & -4 \end{bmatrix}$. Is A diagonalizable?

- A. yes
- B. no
- C. none of the above

Problem 10. Suppose $A \begin{bmatrix} 5 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \\ -1 \end{bmatrix}$. Then an eigenvalue of A is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 11. Suppose the orthogonal projection of $\begin{bmatrix} 13 \\ 3 \\ -4 \end{bmatrix}$ onto $\begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$ is (z_1, z_2, z_3) . Then $z_1 =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above