Math 2550 Matrix Algebra Multiple Choice Exam **Form D**
Oct. 27, 2014

Problem 1. The determinant of a square matrix A is 0 if and only if the equation $Ax = 0$ has an infinite number of solutions.

- A. True
- B. False

Problem 2. $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$

The determinant of the above matrix is

- A. -362880
- B. -40320
- C. -540
- D. -1
- E. 0
- F. 1
- G. 540
- H. 40320
- I. 362880
- J. None of the above

Problem 3. The vector $\vec{b}$ is in $ColA$ if and only if $A\vec{v} = \vec{b}$ has a solution

- A. True
- B. False

Problem 4. If $A = \begin{bmatrix} -1 & 4 \\ -7 & 5 \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, and $AB = I$, the identity matrix, then $b_{11} =$

- A. $\frac{1}{23}$
- B. $\frac{4}{23}$
- C. $\frac{5}{23}$
- D. $\frac{7}{23}$
- E. $-\frac{1}{23}$
- F. $-\frac{4}{23}$
- G. $-\frac{5}{23}$
- H. $-\frac{7}{23}$
- I. 7
- J. None of those above

$$\begin{bmatrix} -1 & 4 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 5/23 & -4/23 \\ 7/23 & -1/23 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Problem 5. Suppose A is a square matrix and $A\vec{x} = \vec{0}$ has a unique solution, then given a vector $\vec{b}$ of the appropriate dimension, $A\vec{x} = \vec{b}$ has

eg $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Problem 6. Given that $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 8 & -4 \\ 12 & -6 \end{bmatrix}$, determine the corresponding eigenvalue.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. None of those above

$$\begin{bmatrix} 8 & -4 \\ 12 & -6 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8-8 \\ 12-12 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 0\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Problem 7. Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ does not span $\mathbb{R}^3$. Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans $\mathbb{R}^3$.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is the zero vector.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is a scalar multiple of another vector in the set.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans $\mathbb{R}^3$ unless $\mathbf{u}_4$ is in span$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans $\mathbb{R}^3$.
- F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ may, but does not have to, span $\mathbb{R}^3$.
- G. none of the above

Problem 8.

not multiples

$\Rightarrow$ l.i $\Rightarrow$ unique soln

Find the null space for $A = \begin{bmatrix} 4 & 7 \\ 7 & 5 \\ -6 & 6 \end{bmatrix}$.

$\Rightarrow \vec{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

What is null(A)?

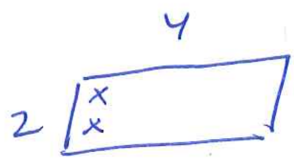
$(3 \times 2)\ \underline{(2 \times 1)}$

- A. span$\left\{ \begin{bmatrix} -7 \\ 4 \end{bmatrix} \right\}$
- B. $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- C. $\mathbb{R}^3$
- D. span$\left\{ \begin{bmatrix} 7 \\ 4 \end{bmatrix} \right\}$
- E. span$\left\{ \begin{bmatrix} 4 \\ 7 \\ -6 \end{bmatrix} \right\}$
- F. span$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$
- G. span$\left\{ \begin{bmatrix} 4 \\ 7 \end{bmatrix} \right\}$
- H. $\mathbb{R}^2$
- I. none of the above

Problem 9. If $\vec{x}_1$ and $\vec{x}_2$ are solutions to $A\vec{x} = \vec{b}$, then $7\vec{x}_1 + 1\vec{x}_2$ is also a solution to $A\vec{x} = \vec{b}$.

- A. True
- B. False

Problem 10. Suppose A is a 2×4 matrix. Then *col* A is a subspace of R^k where $k =$



- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 11. Let A be a matrix with linearly independent columns. Select the best statement.

- A. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it has more rows than columns.
- not l.i B. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it has more columns than rows.
- C. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it is a square matrix.
- D. The equation $A\mathbf{x} = \mathbf{0}$ never has nontrivial solutions.
- E. There is insufficient information to determine if such an equation has nontrivial solutions.
- F. The equation $A\mathbf{x} = \mathbf{0}$ always has nontrivial solutions.
- G. none of the above

Problem 12. Suppose A is a 9×8 matrix. If rank of $A = 6$, then nullity of $A =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

8

9 | 6

$8 - 6 = 2$

Problem 13. Find all values of x for which $\text{rank}(A) = 2$. where A = $\begin{bmatrix} 1 & 2 & 0 & 7 \\ -2 & -2 & x & -5 \\ -3 & -8 & 3 & -30 \end{bmatrix}$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

$$\begin{bmatrix} 1 & 2 & 0 & 7 \\ -2^{+2} & -2^{+4} & x & -5^{+14} \\ -3_{+3} & -8_{+6} & 3 & -30_{+24} \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 7 \\ 0 & 2 & x & 9 \\ 0 & -2 & 3 & -9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 7 \\ 0 & 2 & x & 9 \\ 0 & 0 & x+3 & 0 \end{bmatrix}$$

$x + 3 = 0$

$\Rightarrow x = -3$