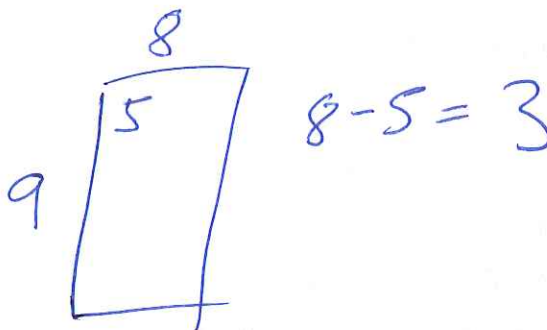


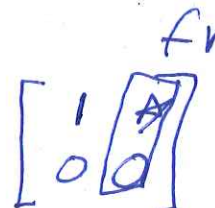
Problem 1. Suppose A is a 9×8 matrix. If rank of $A = 5$, then nullity of $A =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above



Problem 2. Suppose A is a square matrix and $A\vec{x} = \vec{0}$ has an infinite number of solutions, then given a vector \vec{b} of the appropriate dimension, $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above



Problem 3. A vector b is a linear combination of the columns of a matrix A if and only if the equation $Ax = b$ has at least one solution.

- A. True
- B. False

Problem 4.

Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ does not span \mathbb{R}^3 .

Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans \mathbb{R}^3 .
 - B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is the zero vector.
 - C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is a scalar multiple of another vector in the set.
 - D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is in $\text{span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
 - E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans \mathbb{R}^3 .
 - F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ may, but does not have to, span \mathbb{R}^3 .
 - G. none of the above
-

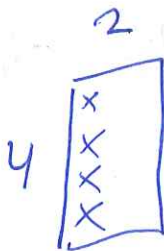
Problem 5. Given that $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is an eigenvector of the matrix $A = \begin{bmatrix} 8 & -3 \\ 18 & -7 \end{bmatrix}$, determine the corresponding eigenvalue.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. None of those above

$$\begin{bmatrix} 8 & -3 \\ 18 & -7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 8-6 \\ 18-14 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Problem 6. Suppose A is a 4×2 matrix. Then $\text{col } A$ is a subspace of R^k where $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above



Problem 7. Let A be a matrix with linearly independent columns. Select the best statement.

- A. The equation $Ax = b$ has a solution for all b precisely when it has more rows than columns. ~~X~~
- B. The equation $Ax = b$ has a solution for all b precisely when it is a square matrix. → pivot in all columns ⇒ pivot in all rows
- C. The equation $Ax = b$ never has a solution for all b . ~~X~~ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- D. The equation $Ax = b$ has a solution for all b precisely when it has more columns than rows. ~~X~~ X_2 not l.i.
- E. There is insufficient information to determine if $Ax = b$ has a solution for all b .
- F. The equation $Ax = b$ always has a solution for all b . ~~X~~
- G. none of the above

Problem 8. If $A = \begin{bmatrix} -1 & 4 \\ -7 & 5 \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, and $AB = I$, the identity matrix, then $b_{11} =$

- A. $\frac{1}{23}$
- B. $\frac{4}{23}$
- C. $\frac{5}{23}$
- D. $\frac{7}{23}$
- E. $-\frac{1}{23}$
- F. $-\frac{4}{23}$
- G. $-\frac{5}{23}$
- H. $-\frac{7}{23}$
- I. 7
- J. None of those above

$$\begin{bmatrix} -1 & 4 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 5/23 & -4/23 \\ 7/23 & -1/23 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Problem 9.

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{bmatrix}$$

The determinant of the above matrix is

- A. -362880
- B. -40320
- C. -540
- D. -1
- E. 0
- F. 1
- G. 540
- H. 40320
- I. 362880
- J. None of the above

Problem 10. If \vec{x}_1 and \vec{x}_2 are solutions to $A\vec{x} = \vec{b}$, then $8\vec{x}_1 - 3\vec{x}_2$ is also a solution to $A\vec{x} = \vec{b}$.

- A. True
- B. False

Problem 11. The determinant of a square matrix A is nonzero if and only if the equation $Ax = 0$ has a unique solution.

- A. True
- B. False

Problem 12. Find the null space for $A = \begin{bmatrix} 1 & -4 \\ -1 & 4 \\ 4 & -16 \end{bmatrix}$. $\sim \begin{bmatrix} 1 & -4 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

What is $\text{null}(A)$?

- A. $\text{span} \left\{ \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} \right\}$

- B. \mathbb{R}^2

- C. \mathbb{R}^3

- D. $\text{span} \left\{ \begin{bmatrix} 4 \\ 1 \end{bmatrix} \right\}$

- E. $\text{span} \left\{ \begin{bmatrix} -16 \\ 4 \end{bmatrix} \right\}$

- F. $\text{span} \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

- G. $\text{span} \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$

- H. none of the above

$$x_1 = 4x_2$$

$$x_2 = x_2$$

$$\Rightarrow \vec{x} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} x_2$$

Check: $\begin{bmatrix} 1 & -4 \\ -1 & 4 \\ 4 & -16 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Problem 13. Find all values of x for which $\text{rank}(A) = 2$, where $A = \begin{bmatrix} 1 & 2 & 0 & 7 \\ -2 & -2 & x & -5 \\ -3 & -8 & 2 & -30 \end{bmatrix}$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

$$\begin{bmatrix} 1 & 2 & 0 & 7 \\ 0 & 2 & x & 9 \\ 0 & -2 & 2 & -9 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 0 & 7 \\ 0 & 2 & x & 9 \\ 0 & 0 & x+2 & 0 \end{bmatrix}$$

$$\begin{aligned} x+2 &= 0 \\ \Rightarrow x &= -2 \end{aligned}$$