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**Problem 1.** Suppose  $A$  is a square matrix and  $A\vec{x} = \vec{0}$  has a unique solution, then given a vector  $\vec{b}$  of the appropriate dimension,  $A\vec{x} = \vec{b}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

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**Problem 2.** Suppose that  $A$  is a  $8 \times 9$  matrix which has a null space of dimension 6. The rank of  $A =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

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**Problem 3.** If  $\vec{x}_1$  and  $\vec{x}_2$  are solutions to  $A\vec{x} = \vec{0}$ , then  $9\vec{x}_1 + 3\vec{x}_2$  is also a solution to  $A\vec{x} = \vec{0}$ .

- A. True
- B. False

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**Problem 4.** Suppose  $A$  is a  $4 \times 3$  matrix. Then  $\text{nul } A$  is a subspace of  $R^k$  where  $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

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**Problem 5.** Let  $\mathbf{u}_4$  be a linear combination of  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ . Select the best statement.

- A.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  is never a linearly dependent set of vectors.
- B.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  is a linearly dependent set of vectors unless one of  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  is the zero vector.
- C.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  is never a linearly independent set of vectors.
- D.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  is always a linearly independent set of vectors.
- E.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  is a linearly dependent set of vectors.
- F.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  could be a linearly dependent or linearly independent set of vectors depending on the vectors chosen.
- G. none of the above

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**Problem 6.** The determinant of a square matrix  $A$  is 0 if and only if the equation  $Ax = 0$  has an infinite number of solutions.

- A. True
- B. False

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**Problem 7.** Let  $A$  be a matrix with linearly independent columns. Select the best statement.

- A. The equation  $A\mathbf{x} = \mathbf{0}$  has nontrivial solutions precisely when it has more rows than columns.
- B. The equation  $A\mathbf{x} = \mathbf{0}$  has nontrivial solutions precisely when it has more columns than rows.
- C. The equation  $A\mathbf{x} = \mathbf{0}$  has nontrivial solutions precisely when it is a square matrix.
- D. The equation  $A\mathbf{x} = \mathbf{0}$  never has nontrivial solutions.
- E. There is insufficient information to determine if such an equation has nontrivial solutions.
- F. The equation  $A\mathbf{x} = \mathbf{0}$  always has nontrivial solutions.
- G. none of the above

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**Problem 8.** Suppose  $A \begin{bmatrix} 5 \\ 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -15 \\ -3 \\ 9 \end{bmatrix}$ . Then an eigenvalue of  $A$  is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

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**Problem 9.** Find the null space for  $A = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 3 \end{bmatrix}$ .

What is  $\text{null}(A)$ ?

- A.  $\text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \right\}$
- B.  $\text{span} \left\{ \begin{bmatrix} -3 \\ -6 \\ 1 \end{bmatrix} \right\}$
- C.  $\mathbb{R}^3$
- D.  $\mathbb{R}^2$
- E.  $\text{span} \left\{ \begin{bmatrix} -6 \\ -3 \\ 1 \end{bmatrix} \right\}$
- F.  $\text{span} \left\{ \begin{bmatrix} -6 \\ -3 \end{bmatrix} \right\}$
- G.  $\text{span} \left\{ \begin{bmatrix} -3 \\ -6 \end{bmatrix} \right\}$
- H. none of the above

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**Problem 10.**  $A = \begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}$ . If  $A^2 = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ , find  $b_{12}$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. None of those above

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**Problem 11.** Find the volume of the parallelepiped determined by vectors  $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$ , and  $\begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix}$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. None of those above

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**Problem 12.** The vector  $\vec{b}$  is in  $ColA$  if and only if  $A\vec{v} = \vec{b}$  has a solution

- A. True
- B. False

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**Problem 13.** The vectors  $v = \begin{bmatrix} -2 \\ -2 \\ 2 \end{bmatrix}$ ,  $u = \begin{bmatrix} -3 \\ 7 \\ -17 \end{bmatrix}$ , and  $w = \begin{bmatrix} 3 \\ -3 \\ 8+k \end{bmatrix}$  are linearly independent if and only if  $k \neq$  \_\_\_\_\_.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above