Math 2550 Matrix Algebra Multiple Choice Exam **Form A**
Oct. 27, 2014

Problem 1. The determinant of a square matrix A is nonzero if and only if the equation $Ax = 0$ has a unique solution.

$\neq 0$

- A. True
- B. False

Problem 2. Find the volume of the parallelepiped determined by vectors $\begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix}$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. None of those above

$$\begin{vmatrix} 2 & 0 & 0 \\ 0 & -2 & -5 \\ 2 & 0 & 1 \end{vmatrix} = 2\begin{vmatrix} -2 & -5 \\ 0 & 1 \end{vmatrix}$$

$$= 2(-2-0) = -4$$

Problem 3. A vector b is a linear combination of the columns of a matrix A if and only if the equation $Ax = b$ has at least one solution.

- A. True
- B. False

Problem 4. $A = \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix}$. If $A^2 = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$, find b_{12}

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. None of those above

$$\begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} & -2-1 \\ & \end{bmatrix}$$

Problem 5. Suppose A is a square matrix and $A\vec{x} = \vec{0}$ has an infinite number of solutions, then given a vector $\vec{b}$ of the appropriate dimension, $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

$$\begin{bmatrix} 1 & * \\ 0 & 0 \end{bmatrix}$$

Problem 6. Suppose $A \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ -12 \\ 4 \end{bmatrix}$. Then an eigenvalue of A is

x 4

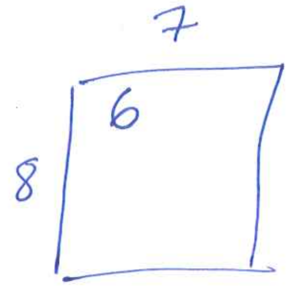
- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Problem 7. Let $\mathbf{u}_4$ be a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. Select the best statement.

- A. $\cancel{\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}}$ is never a linearly dependent set of vectors.
- B. $\cancel{\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}}$ is a linearly dependent set of vectors unless one of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is the zero vector.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is never a linearly independent set of vectors.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.
- E. $\cancel{\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}}$ is a linearly dependent set of vectors.
- F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly dependent or linearly independent set of vectors depending on the vectors chosen.
- G. none of the above

Problem 8. Suppose that A is an 8×7 matrix which has a null space of dimension 6. The rank of A=

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above



$7-6=1$

Problem 9. Find the null space for $A = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 3 \end{bmatrix}$. What is null$(A)$?

$\begin{bmatrix}-6\\-3\\1\end{bmatrix} = \begin{bmatrix}-6+6\\-3+3\end{bmatrix} = \begin{bmatrix}0\\0\end{bmatrix}$ ✓

$x_1 = -6x_3$
$x_2 = -3x_3$
$x_3 = x_3$
$\Rightarrow \vec{x} = \begin{bmatrix}-6\\-3\\1\end{bmatrix} x_3$

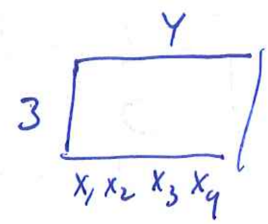
- A. span$\left\{ \begin{bmatrix} 1 \\ 0 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix} \right\}$
- B. span$\left\{ \begin{bmatrix} -3 \\ -6 \\ 1 \end{bmatrix} \right\}$
- C. $\mathbb{R}^3$
- D. $\mathbb{R}^2$
- E. span$\left\{ \begin{bmatrix} -6 \\ -3 \\ 1 \end{bmatrix} \right\}$
- F. span$\left\{ \begin{bmatrix} -6 \\ -3 \end{bmatrix} \right\}$
- G. span$\left\{ \begin{bmatrix} -3 \\ -6 \end{bmatrix} \right\}$
- H. none of the above

Problem 10. If $\vec{x}_1$ and $\vec{x}_2$ are solutions to $A\vec{x} = \vec{0}$, then $7\vec{x}_1 + 1\vec{x}_2$ is also a solution to $A\vec{x} = \vec{0}$.

- A. True
- B. False

Problem 11. Suppose A is a 3×4 matrix. Then *nul* A is a subspace of R^k where $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above



Problem 12. Let A be a matrix with linearly independent columns.
Select the best statement.

- $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ~~A.~~ The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$ precisely when it has more rows than columns.
- B. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$ precisely when it is a square matrix. pivot in each column $\Rightarrow$ pivot in each row.
- C. The equation $A\mathbf{x} = \mathbf{b}$ never has a solution for all $\mathbf{b}$.
- not li ~~D.~~ The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$ precisely when it has more columns than rows.
- E. There is insufficient information to determine if $A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$.
- F. The equation $A\mathbf{x} = \mathbf{b}$ always has a solution for all $\mathbf{b}$.
- G. none of the above

Problem 13. The vectors $v = \begin{bmatrix} -2 \\ -2 \\ 2 \end{bmatrix}$, $u = \begin{bmatrix} -3 \\ 7 \\ -17 \end{bmatrix}$, and $w = \begin{bmatrix} 3 \\ -3 \\ 6+k \end{bmatrix}$. are linearly independent if and only if $k \neq$ ____.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

$$\begin{bmatrix} -2 & -3 & 3 \\ -2^{+2} & 7^{+3} & -3^{-3} \\ 2 & -17 & 6+K \end{bmatrix}$$

$$\sim \begin{bmatrix} -2 & -3 & 3 \\ 0 & 10 & -6 \\ 0 & -20^{+20} & 9+K \end{bmatrix}$$

$$\sim \begin{bmatrix} -2 & -3 & 3 \\ 0 & 10 & -6 \\ 0 & 0 & K-3 \end{bmatrix}$$

$K-3=0 \Rightarrow$

$K=3$