

1. (1 pt) Library/Rochester/setLinearAlgebra4InverseMatrix-
/ur.Ch2.1.4.pg

Are the following matrices invertible? Enter "Y" or "N".
You must get all of the answers correct to receive credit.

- 1. $\begin{bmatrix} -4 & 0 \\ 0 & 0 \end{bmatrix}$
 —2. $\begin{bmatrix} 7 & 0 \\ 0 & -6 \end{bmatrix}$
 —3. $\begin{bmatrix} 2 & 0 \\ -4 & 0 \end{bmatrix}$
 —4. $\begin{bmatrix} -2 & 4 \\ -4 & -5 \end{bmatrix}$

Correct Answers:

- N
- Y
- N
- Y

2. (1 pt) Library/Rochester/setLinearAlgebra4InverseMatrix-
/ur.la.4.2.pg

The matrix $\begin{bmatrix} 2 & -6 \\ -5 & k \end{bmatrix}$ is invertible if and only if $k \neq$ ____.

Correct Answers:

- 15

3. (1 pt) Library/Rochester/setLinearAlgebra9Dependence-
/ur.la.9.7.pg

The vectors

$$v = \begin{bmatrix} -4 \\ -9 \\ -5 \end{bmatrix}, u = \begin{bmatrix} -3 \\ -9 \\ -12 + k \end{bmatrix}, \text{ and } w = \begin{bmatrix} 3 \\ 11 \\ 8 \end{bmatrix}.$$

are linearly independent if and only if $k \neq$ ____.

Correct Answers:

- 6

4. (1 pt) Library/Rochester/setLinearAlgebra9Dependence-
/ur.la.9.10.pg

Express the vector $v = \begin{bmatrix} 14 \\ -26 \end{bmatrix}$ as a linear combination of

$$x = \begin{bmatrix} 4 \\ -5 \end{bmatrix} \text{ and } y = \begin{bmatrix} -5 \\ 2 \end{bmatrix}.$$

$$v = \underline{\hspace{1cm}} x + \underline{\hspace{1cm}} y.$$

Correct Answers:

- 6
- 2

5. (1 pt) Library/TCNJ/TCNJ_BasesLinearlyIndependentSet-
/problem5.pg

Let W_1 be the set: $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$

Determine if W_1 is a basis for \mathbb{R}^3 and check the correct answer(s) below.

- A. W_1 is not a basis because it does not span \mathbb{R}^3 .
- B. W_1 is a basis.
- C. W_1 is not a basis because it is linearly dependent.

Let W_2 be the set: $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$

Determine if W_2 is a basis for \mathbb{R}^3 and check the correct answer(s) below.

- A. W_2 is not a basis because it is linearly dependent.
- B. W_2 is not a basis because it does not span \mathbb{R}^3 .
- C. W_2 is a basis.

Correct Answers:

- B
- AB

6. (1 pt) Library/TCNJ/TCNJ_LinearIndependence/problem3.pg

If k is a real number, then the vectors $(1, k), (k, k + 56)$ are linearly independent precisely when

$$k \neq a, b,$$

where $a =$ ____, $b =$ ____, and $a < b$.

Correct Answers:

- -7
- 8

7. (1 pt) Library/TCNJ/TCNJ_LinearSystems/problem1.pg

Determine whether the following system has no solution, an infinite number of solutions or a unique solution.

$$\begin{array}{lcl} 7x & -7y & = 7 \\ \boxed{?} 1. & 2x & -7y = 2 \\ & -11x & +21y = -11 \\ & 7x & -7y = 7 \\ \boxed{?} 2. & 2x & -7y = 2 \\ & -11x & +21y = -13 \end{array}$$

$$\begin{array}{rcl} 16x + 12y & = & -4 \\ ?3. \quad 12x + 9y & = & -3 \\ -28x - 21y & = & 7 \end{array}$$

Correct Answers:

- Unique Solution
- No Solution
- Infinite Solutions

8. (1 pt) Library/TCNJ/TCNJ_LinearSystems/problem2.pg

Determine whether the following system has no solution, an infinite number of solutions or a unique solution.

$$\begin{array}{rcl} ?1. \quad -10x + 15y + 20z & = & 0 \\ -8x + 12y + 16z & = & 0 \\ ?2. \quad 5x + 8y - 7z & = & 8 \\ -3x + 9y - 2z & = & 9 \\ ?3. \quad -10x + 15y + 20z & = & 0 \\ -8x + 12y + 16z & = & 3 \end{array}$$

Correct Answers:

- Infinite Solutions
- Infinite Solutions
- No Solution

9. (1 pt) Library/TCNJ/TCNJ_LinearSystems/problem3.pg

Give a geometric description of the following systems of equations

$$\begin{array}{rcl} ?1. \quad -20x + 25y & = & -5 \\ -8x + 10y & = & -2 \\ 24x - 30y & = & 6 \\ 3x - 5y & = & 7 \\ ?2. \quad 6x + 3y & = & 3 \\ -3x - 21y & = & 16 \\ 3x - 5y & = & 7 \\ ?3. \quad 6x + 3y & = & 3 \\ -3x - 21y & = & 15 \end{array}$$

Correct Answers:

- Three identical lines
- Three non-parallel lines with no common intersection
- Three lines intersecting at a single point

10. (1 pt) Library/TCNJ/TCNJ_LinearSystems/problem4.pg

Give a geometric description of the following system of equations

$$\begin{array}{rcl} ?1. \quad 2x + 4y - 6z & = & 12 \\ -x + 5y - 9z & = & 1 \\ ?2. \quad 2x + 4y - 6z & = & 12 \\ -3x - 6y + 9z & = & 16 \\ ?3. \quad 2x + 4y - 6z & = & -12 \\ -3x - 6y + 9z & = & 18 \end{array}$$

Correct Answers:

- Two planes intersecting in a line
- Two parallel planes
- Two planes that are the same

11. (1 pt) Library/TCNJ/TCNJ_LinearSystems/problem11.pg

Give a geometric description of the following systems of equations.

$$\begin{array}{rcl} ?1. \quad -6x - 6y & = & 0 \\ 15x + 15y & = & 1 \\ ?2. \quad -6x - 6y & = & 0 \\ 15x + 15y & = & 0 \\ ?3. \quad -3x + 9y & = & 3 \\ 4x - 6y & = & -5 \end{array}$$

Correct Answers:

- Two parallel lines
- Two lines that are the same
- Two lines intersecting in a point

12. (1 pt) Library/TCNJ/TCNJ_MatrixEquations/problem4.pg

Let $A = \begin{bmatrix} -5 & 1 & 3 \\ 3 & -3 & 2 \\ 3 & -1 & 3 \end{bmatrix}$ and $x = \begin{bmatrix} -4 \\ -5 \\ 1 \end{bmatrix}$.

?1. What does Ax mean?

Correct Answers:

- Linear combination of the columns of A

13. (1 pt) Library/TCNJ/TCNJ_MatrixEquations/problem13.pg

Do the following sets of vectors span \mathbb{R}^3 ?

$$\begin{array}{rcl} ?1. \quad \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -7 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} -19 \\ 12 \\ 8 \end{bmatrix} \\ ?2. \quad \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ -1 \end{bmatrix}, \begin{bmatrix} 6 \\ 9 \\ -2 \end{bmatrix}, \begin{bmatrix} 14 \\ 21 \\ -3 \end{bmatrix} \\ ?3. \quad \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix} \\ ?4. \quad \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \\ 4 \end{bmatrix} \end{array}$$

Correct Answers:

- Yes
- No
- No
- No

14. (1 pt) Library/TCNJ/TCNJ.VectorEquations/problem5.pg
Let $H = \text{span}\{u, v\}$. For each of the following sets of vectors determine whether H is a line or a plane.

- ? 1. $u = \begin{bmatrix} -1 \\ -3 \\ -2 \end{bmatrix}, v = \begin{bmatrix} 2 \\ 8 \\ 6 \end{bmatrix},$
 ? 2. $u = \begin{bmatrix} -4 \\ -5 \\ 5 \end{bmatrix}, v = \begin{bmatrix} 15 \\ 20 \\ -22 \end{bmatrix},$
 ? 3. $u = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$
 ? 4. $u = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 6 \\ -4 \\ 2 \end{bmatrix},$

Correct Answers:

- Plane
- Plane
- Line
- Line

15. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4-/2.2.8.pg

Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$.

Is \mathbf{b} in the span of \mathbf{a}_1 ?

- A. Yes, \mathbf{b} is in the span.
- B. No, \mathbf{b} is not in the span.
- C. We cannot tell if \mathbf{b} is in the span.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.

$\mathbf{b} = \underline{\hspace{1cm}} \mathbf{a}_1$

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

From the first component we see $3 = 3 * 1$.

From the second component we see $6 = 3 * 2$.

Thus $\mathbf{b} = 3\mathbf{a}_1$ is in the span of \mathbf{a}_1 .

Correct Answers:

- A
- 3

16. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4-/2.2.31.pg

Let $A = \begin{bmatrix} 3 & 6 \\ -4 & 17 \\ -7 & 5 \end{bmatrix}$.

We want to determine if the system $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b} \in \mathbb{R}^3$.

Select the best answer.

- A. There is not a solution for every $\mathbf{b} \in \mathbb{R}^3$ since $2 < 3$.
- B. There is a solution for every $\mathbf{b} \in \mathbb{R}^3$ since $2 < 3$
- C. There is a solution for every $\mathbf{b} \in \mathbb{R}^3$ but we need to row reduce A to show this.
- D. There is a not solution for every $\mathbf{b} \in \mathbb{R}^3$ but we need to row reduce A to show this.
- E. We cannot tell if there is a solution for every $\mathbf{b} \in \mathbb{R}^3$.

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

There is not a solution for every $\mathbf{b} \in \mathbb{R}^3$ since $2 < 3$.

Correct Answers:

- A

17. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4-/2.2.56.pg

What conditions on a matrix A insures that $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} in \mathbb{R}^n ?

Select the best statement. (The best condition should work with any positive integer n .)

- A. The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as no column of A is a scalar multiple of another column.
- B. The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as the columns of A do not include the zero column.
- C. There is no easy test to determine if the equation will have a solution for all \mathbf{b} in \mathbb{R}^n .
- D. The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as the columns of A span \mathbb{R}^n .
- E. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

The equation will have a solution for all \mathbf{b} in \mathbb{R}^n as long as the columns of A span \mathbb{R}^n .

Correct Answers:

- D

18. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4-/2.2.57.pg

Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ spans \mathbb{R}^3 .

Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is the zero vector.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans \mathbb{R}^3 .

- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is a scalar multiple of another vector in the set.
- D. There is no easy way to determine if $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 .
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans \mathbb{R}^3 .
- F. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

The span of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a subset of the span of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$, so $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans \mathbb{R}^3 .

Correct Answers:

- B

19. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.2.58.pg

Assume $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ does not span \mathbb{R}^3 .
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ never spans \mathbb{R}^3 .
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is a scalar multiple of another vector in the set.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ spans \mathbb{R}^3 unless \mathbf{u}_4 is the zero vector.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ may, but does not have to, span \mathbb{R}^3 .
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ always spans \mathbb{R}^3 .
- F. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ may, but does not have to, span \mathbb{R}^3 .

Correct Answers:

- D

20. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.3.40.pg

Let \mathbf{S} be a set of m vectors in \mathbb{R}^n with $m > n$.
Select the best statement.

- A. The set \mathbf{S} is linearly independent.
- B. The set \mathbf{S} is linearly independent, as long as it does not include the zero vector.
- C. The set \mathbf{S} is linearly dependent.
- D. The set \mathbf{S} could be either linearly dependent or linearly independent, depending on the case.
- E. The set \mathbf{S} is linearly independent, as long as no vector in \mathbf{S} is a scalar multiple of another vector in the set.
- F. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

By theorem 2.13, a linearly independent set in \mathbb{R}^n can contain no more than n vectors.

Correct Answers:

- C

21. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.3.41.pg

Let A be a matrix with more rows than columns.
Select the best statement.

- A. The columns of A must be linearly dependent.
- B. The columns of A are linearly independent, as long as no column is a scalar multiple of another column in A .
- C. The columns of A are linearly independent, as long as they does not include the zero vector.
- D. The columns of A could be either linearly dependent or linearly independent depending on the case.
- E. The columns of A must be linearly independent.
- F. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

The zero matrix is an example where the columns are linearly dependent. The matrix where the top square portion is the identity matrix and the portion below that is all zeros is an example where the columns are linearly independent.

The columns of A could be either linearly dependent or linearly independent depending on the case.

Correct Answers:

- D

22. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.3.42.pg

Let A be a matrix with more columns than rows.
Select the best statement.

- A. The columns of A are linearly independent, as long as they does not include the zero vector.
- B. The columns of A are linearly independent, as long as no column is a scalar multiple of another column in A .
- C. The columns of A could be either linearly dependent or linearly independent depending on the case.
- D. The columns of A must be linearly dependent.
- E. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

Since there are more columns than rows, when we row reduce the matrix not all columns can have a leading 1.

The columns of A must be linearly dependent.

Correct Answers:

- D

23. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.3.46.pg

Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ be a linearly dependent set of vectors. Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly independent set of vectors unless \mathbf{u}_4 is a linear combination of other vectors in the set.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is a linearly independent set of vectors unless $\mathbf{u}_4 = \mathbf{0}$.
- F. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

If the zero vector is a nontrivial linear combination of a vectors in a smaller set, then it is also a nontrivial combination of vectors in a bigger set containing those vectors.

$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly dependent set of vectors.

Correct Answers:

- D

24. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/2.3.47.pg

Let $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ be a linearly independent set of vectors. Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is never a linearly independent set of vectors.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is always a linearly independent set of vectors.
- D. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

If the zero vector cannot be written as a nontrivial linear combination of a vectors in a smaller set, then it is also not a nontrivial combination of vectors in a proper subset of those vectors.

$\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is always a linearly independent set of vectors.

Correct Answers:

- C

25. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/3.3.42.pg

A must be a square matrix to be invertible. ☐

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION: True, since

$$(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}I_nB = B^{-1}B = I_n.$$

Correct Answers:

- True

26. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/4.1.22.pg

Find the null space for $A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$.

What is $\text{null}(A)$?

- A. $\text{span}\left\{\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right\}$
- B. $\text{span}\left\{\begin{bmatrix} -2 \\ 1 \end{bmatrix}\right\}$
- C. \mathbb{R}^2
- D. $\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$
- E. $\text{span}\left\{\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right\}$
- F. $\text{span}\left\{\begin{bmatrix} -3 \\ 1 \end{bmatrix}\right\}$
- G. $\text{span}\left\{\begin{bmatrix} 3 \\ 1 \end{bmatrix}\right\}$
- H. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

A row reduces to the identity matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has only the trivial solution $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$,

and thus, $\text{null}(A) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Correct Answers:

- D

27. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4-
/4.1.27.pg

Find the null space for $A = \begin{bmatrix} 3 & -4 \\ 2 & 2 \\ 1 & -7 \end{bmatrix}$.

What is $\text{null}(A)$?

- A. \mathbb{R}^2
- B. $\text{span}\left\{\begin{bmatrix} 3 \\ -4 \end{bmatrix}\right\}$
- C. $\text{span}\left\{\begin{bmatrix} +4 \\ 3 \end{bmatrix}\right\}$
- D. $\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$
- E. $\text{span}\left\{\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}\right\}$
- F. \mathbb{R}^3
- G. $\text{span}\left\{\begin{bmatrix} -4 \\ 3 \end{bmatrix}\right\}$
- H. $\text{span}\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$
- I. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

A is row reduces to $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$. The basis of the null space has

one element for each column without a leading one in the row reduced matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has a zero dimensional null space,

and $\text{null}(A)$ is the zero vector $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Correct Answers:

- D

28. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4-
/4.1.28.pg

Find the null space for $A = \begin{bmatrix} 1 & -2 \\ 4 & -8 \\ -7 & 14 \end{bmatrix}$.

What is $\text{null}(A)$?

- A. $\text{span}\left\{\begin{bmatrix} 1 \\ 4 \\ -7 \end{bmatrix}\right\}$
- B. \mathbb{R}^2

- C. $\text{span}\left\{\begin{bmatrix} +2 \\ 1 \end{bmatrix}\right\}$
- D. $\text{span}\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$
- E. $\text{span}\left\{\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right\}$
- F. $\text{span}\left\{\begin{bmatrix} 14 \\ -7 \end{bmatrix}\right\}$
- G. \mathbb{R}^3
- H. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

A is row reduces to $\begin{bmatrix} 1 & -2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$. The basis of the null space

has one element for each column without a leading one in the row reduced matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has a one dimensional null space,

and $\text{null}(A)$ is the subspace generated by $\begin{bmatrix} +2 \\ 1 \end{bmatrix}$.

Correct Answers:

- C

29. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4-
/4.1.30.pg

Find the null space for $A = \begin{bmatrix} 3 & -1 & 3 \\ -6 & -3 & -21 \\ 2 & 2 & 10 \end{bmatrix}$.

What is $\text{null}(A)$?

- A. $\text{span}\left\{\begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -6 \\ -3 \\ -21 \end{bmatrix}\right\}$
- B. \mathbb{R}^3
- C. $\text{span}\left\{\begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}\right\}$
- D. $\text{span}\left\{\begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix}\right\}$
- E. $\text{span}\left\{\begin{bmatrix} 3 \\ -6 \\ 2 \end{bmatrix}\right\}$
- F. $\text{span}\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$
- G. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

A is row reduces to $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{bmatrix}$. The basis of the null space

has one element for each column without a leading one in the row reduced matrix.

Thus $Ax = \mathbf{0}$ has a one dimensional null space,

and $\text{null}(A)$ is the subspace generated by $\begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix}$.

Correct Answers:

- D

30. (1 pt) Library/WHFreeman/Holt_linear_algebra/Chaps.1-4/4.3.47.pg

Indicate whether the following statement is true or false.

☐ 1. If A and B are equivalent matrices, then $\text{col}(A) = \text{col}(B)$.

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION:

FALSE. Consider $A = \begin{bmatrix} 2 & 3 & 4 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 0 & 0 \end{bmatrix}$

Correct Answers:

- F

31. (1 pt) Library/maCalcDB/setLinearAlgebra3Matrices/ur_la.3.6.pg

If A and B are 3×6 matrices, and C is a 7×3 matrix, which of the following are defined?

- A. BC
- B. BA^T
- C. C^T
- D. CA
- E. $B + A$
- F. $A + C$

Correct Answers:

- BCDE

32. (1 pt) Library/maCalcDB/setLinearAlgebra4InverseMatrix/ur_la.4.8.pg

Determine which of the formulas hold for all invertible $n \times n$ matrices A and B

- A. $A + B$ is invertible
- B. $(AB)^{-1} = A^{-1}B^{-1}$
- C. $AB = BA$
- D. $(A + B)^2 = A^2 + B^2 + 2AB$
- E. A^6 is invertible

- F. $(I_n + A)(I_n + A^{-1}) = 2I_n + A + A^{-1}$

Correct Answers:

- EF

33. (1 pt) UI/Fall14/lin_span2.pg

Which of the following sets of vectors span R^3 ?

- A. $\begin{bmatrix} -3 \\ -8 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -6 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix}$
- B. $\begin{bmatrix} -1 \\ -8 \end{bmatrix}, \begin{bmatrix} -7 \\ 1 \end{bmatrix}$
- C. $\begin{bmatrix} -5 \\ 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \\ -9 \end{bmatrix}, \begin{bmatrix} -12 \\ -1 \\ 17 \end{bmatrix}$
- D. $\begin{bmatrix} 8 \\ -4 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \end{bmatrix}$
- E. $\begin{bmatrix} 8 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ -9 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \end{bmatrix}$
- F. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 9 \end{bmatrix}$

Which of the following sets of vectors are linearly independent?

- A. $\begin{bmatrix} -3 \\ -8 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -6 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix}$
- B. $\begin{bmatrix} -1 \\ -8 \end{bmatrix}, \begin{bmatrix} -7 \\ 1 \end{bmatrix}$
- C. $\begin{bmatrix} -5 \\ 4 \\ 8 \end{bmatrix}, \begin{bmatrix} 7 \\ 5 \\ -9 \end{bmatrix}, \begin{bmatrix} -12 \\ -1 \\ 17 \end{bmatrix}$
- D. $\begin{bmatrix} 8 \\ -4 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \end{bmatrix}$
- E. $\begin{bmatrix} 8 \\ 7 \end{bmatrix}, \begin{bmatrix} 5 \\ -9 \end{bmatrix}, \begin{bmatrix} -4 \\ 2 \end{bmatrix}$
- F. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 9 \end{bmatrix}$

Correct Answers:

- A
- AB

34. (1 pt) UI/Fall14/lin_span.pg

Let $A = \begin{bmatrix} 14 \\ 3 \\ 32 \end{bmatrix}$, $B = \begin{bmatrix} -2 \\ 1 \\ -6 \end{bmatrix}$, and $C = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix}$.

Which of the following best describes the span of the above 3 vectors?

- A. 0-dimensional point in R^3

- B. 1-dimensional line in R^3
- C. 2-dimensional plane in R^3
- D. R^3

Determine whether or not the three vectors listed above are linearly independent or linearly dependent.

- A. linearly dependent
- B. linearly independent

If they are linearly dependent, determine a non-trivial linear relation. Otherwise, if the vectors are linearly independent, enter 0's for the coefficients, since that relationship **always** holds.

$$\text{---}A + \text{---}B + \text{---}C = 0.$$

Correct Answers:

- C
- A
- -1; -2; -5

35. (1 pt) local/Library/Rochester/setLinearAlgebra3Matrices-ur.1a.3.14.pg

Find the ranks of the following matrices.

$$\text{rank} \begin{bmatrix} 0 & 6 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = \text{---}$$

$$\text{rank} \begin{bmatrix} 5 & -7 \\ 10 & -14 \end{bmatrix} = \text{---}$$

$$\text{rank} \begin{bmatrix} 7 & 1 & -7 \\ 0 & -7 & 0 \\ -5 & 0 & 5 \end{bmatrix} = \text{---}$$

Correct Answers:

- 3
- 1
- 2

36. (1 pt) local/Library/TCNJ/TCNJ_BasesLinearlyIndependentSet/3.pg

Check the true statements below:

- A. If B is an echelon form of a matrix A , then the pivot columns of B form a basis for $\text{Col}A$.
- B. A basis is a spanning set that is as large as possible.
- C. The column space of a matrix A is the set of solutions of $Ax = b$.
- D. The columns of an invertible $n \times n$ matrix form a basis for \mathbb{R}^n .
- E. If $H = \text{Span}\{b_1, \dots, b_p\}$, then $\{b_1, \dots, b_p\}$ is a basis for H .

Correct Answers:

- D

37. (1 pt) local/Library/TCNJ/TCNJ_LinearSystems/problem6.pg
Give a geometric description of the following systems of equations

$$\begin{aligned} \text{?}1. \quad & \begin{aligned} -5x - 15y - 36z &= 7 \\ 5x + 14y + 33z &= 3 \\ 5x + 15y + 35z &= 6 \end{aligned} \\ \text{?}2. \quad & \begin{aligned} 3x + 5y + 3z &= -5 \\ -x + 2y + 4z &= 2 \\ -11x - 33y - 31z &= 17 \end{aligned} \\ \text{?}3. \quad & \begin{aligned} 3x + 5y + 3z &= -5 \\ -x + 2y + 4z &= 2 \\ -11x - 33y - 31z &= 20 \end{aligned} \\ \text{?}4. \quad & \begin{aligned} -3x + 3y - 6z &= 3 \\ 4x - 4y + 8z &= -4 \\ 6x - 6y + 12z &= -6 \end{aligned} \end{aligned}$$

Hint: (Instructor hint preview: show the student hint after 1 attempts. The current number of attempts is 0.)

Reduce the augmented matrix and solve for it. If it has unique solutions, three planes intersect at a point; no solutions indicates no common intersection; one free variable shows intersection on a line; two free variables means identical planes.

Correct Answers:

- Three planes intersecting at a point
- Three planes intersecting in a line
- Three planes with no common intersection
- Three identical planes

38. (1 pt) local/Library/UI/2.3.49.pg

Let \mathbf{u}_4 be a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$.
Select the best statement.

- A. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is always a linearly independent set of vectors.
- B. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is never a linearly dependent set of vectors.
- C. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is never a linearly independent set of vectors.
- D. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ could be a linearly dependent or linearly independent set of vectors depending on the vectors chosen.
- E. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linearly dependent set of vectors.
- F. $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linearly dependent set of vectors unless one of $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is the zero vector.
- G. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

$$\begin{aligned} \text{If } \mathbf{u}_4 &= a_1\mathbf{u}_1 + a_2\mathbf{u}_2 + a_3\mathbf{u}_3, \text{ then} \\ 0 &= a_1\mathbf{u}_1 + a_2\mathbf{u}_2 + a_3\mathbf{u}_3 - \mathbf{u}_4 \end{aligned}$$

” $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$ is never a linearly independent set of vectors.”

Correct Answers:

- C

39. (1 pt) local/Library/UI/4.1.23.pg

Find the null space for $A = \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 3 \end{bmatrix}$.

What is $\text{null}(A)$?

- A. \mathbb{R}^3
- B. $\text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ +6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}\right\}$
- C. $\text{span}\left\{\begin{bmatrix} -3 \\ +6 \end{bmatrix}\right\}$
- D. \mathbb{R}^2
- E. $\text{span}\left\{\begin{bmatrix} +6 \\ -3 \\ 1 \end{bmatrix}\right\}$
- F. $\text{span}\left\{\begin{bmatrix} +6 \\ -3 \end{bmatrix}\right\}$
- G. $\text{span}\left\{\begin{bmatrix} -3 \\ +6 \\ 1 \end{bmatrix}\right\}$
- H. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

A is row reduced. The basis of the null space has one element for each column without a leading one in the row reduced matrix.

Thus $A\mathbf{x} = \mathbf{0}$ has a one dimensional null space,

and thus, $\text{null}(A)$ is the subspace generated by $\begin{bmatrix} 1-6 \\ 13 \\ 1 \end{bmatrix}$.

Correct Answers:

- E

40. (1 pt) local/Library/UI/Fall14/HW7.4.pg

Determine if the subset of \mathbb{R}^2 consisting of vectors of the form $\begin{bmatrix} a \\ b \end{bmatrix}$, where a and b are integers, is a subspace.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

The vector $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is included in the set, but the vector $(1/2) * \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \end{bmatrix}$ is not included in the set.

Correct Answers:

- A
- A
- B
- B

41. (1 pt) local/Library/UI/Fall14/HW7.5.pg

Determine if the subset of \mathbb{R}^3 consisting of vectors of the form $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$, where $a \geq 0$, $b \geq 0$, and $c \geq 0$ is a subspace.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

The vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is included in the set, but the vector $(-1) * \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$ is not included in the set.

Correct Answers:

- A
- A
- B
- B

42. (1 pt) local/Library/UI/Fall14/HW7.6.pg

If A is an $n \times n$ matrix and $\mathbf{b} \neq \mathbf{0}$ in \mathbb{R}^n , then consider the set of solutions to $A\mathbf{x} = \mathbf{b}$.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

$A\mathbf{0} = \mathbf{0} \neq \mathbf{b}$, so the zero vector is not in the set and it is not a subspace.

Correct Answers:

- B
- B
- B
- B

43. (1 pt) local/Library/UI/Fall14/HW7.11.pg

Find all values of x for which $\text{rank}(A) = 2$.

$$A = \begin{bmatrix} 2 & 2 & 0 & 5 \\ 4 & 7 & x & 19 \\ -6 & -9 & -3 & -24 \end{bmatrix}$$

$x =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION:

Row reduce A to get:

$$\begin{bmatrix} 2 & 2 & 0 & 5 \\ 4 & 7 & x & 19 \\ -6 & -9 & -3 & -24 \end{bmatrix} \sim \begin{bmatrix} 2 & 2 & 0 & 5 \\ 0 & 3 & x & 9 \\ 0 & -3 & -3 & -9 \end{bmatrix}$$

Since two pivots are needed, $x = 3$

Correct Answers:

- H

44. (1 pt) local/Library/UI/Fall14/HW7.12.pg

Suppose that A is a 5×6 matrix which has a null space of dimension 6. The rank of $A =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

Using the Rank-Nullity theorem, if the dimensions of A is $n \times m$, $\text{rank}(A) = m - \text{nullity}(A) = 6 - 6 = 0$

Correct Answers:

- E

45. (1 pt) local/Library/UI/Fall14/HW7.25.pg

Indicate whether the following statement is true or false?

If $S = \text{span}\{u_1, u_2, u_3\}$, then $\dim(S) = 3$.

- A. True
- B. False

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION:

FALSE. For example, suppose

$$S = \text{span}\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\},$$

then $\dim(S) < 3$

Correct Answers:

- B

46. (1 pt) local/Library/UI/Fall14/HW7.27.pg

Determine the rank and nullity of the matrix.

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ 5 & 2 & 1 & -4 \\ -1 & -4 & -1 & 6 \\ -8 & -5 & -2 & 9 \end{bmatrix}$$

The rank of the matrix is

- A. -4
- B. -3
- C. -2

- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

The nullity of the matrix is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION:

When reduced to row-echelon form, there are two non-zero rows, so the rank of the matrix is 2 and the nullity is 2.

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ 5 & 2 & 1 & -4 \\ -1 & -4 & -1 & 6 \\ -8 & -5 & -2 & 9 \end{bmatrix} \sim \begin{bmatrix} 1 & 4 & 1 & -6 \\ 0 & 9 & 2 & -13 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Correct Answers:

- G
- G

47. (1 pt) local/Library/UI/Fall14/HW8.2.pg

Evaluate the following 3×3 determinant. Use the properties of determinants to your advantage.

$$\begin{vmatrix} -8 & 0 & -4 \\ -1 & 0 & 10 \\ -4 & 0 & 3 \end{vmatrix}$$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Does the matrix have an inverse?

- A. No
- B. Yes

Correct Answers:

- E
- A

48. (1 pt) local/Library/UI/Fall14/HW8.3.pg

Given the matrix $\begin{bmatrix} -4 & 0 & 1 \\ 0 & -2 & 3 \\ 3 & 0 & 0 \end{bmatrix}$

(a) find its determinant

- A. 6
- B. -5
- C. -4
- D. -2
- E. -1
- F. 0
- G. 1
- H. 3
- I. 5
- J. 7
- K. None of those above

(b) Does the matrix have an inverse?

- A. No
- B. Yes

Correct Answers:

- A
- B

49. (1 pt) local/Library/UI/Fall14/HW8.4.pg

If A and B are 4×4 matrices, $\det(A) = 1$, $\det(B) = 2$, then $\det(AB) =$

- A. -15
- B. 2
- C. -11
- D. -8
- E. -5
- F. 0
- G. 3
- H. 6
- I. 8
- J. 12
- K. None of those above

$$\det(-3A) =$$

- A. -40
- B. 81
- C. -28
- D. -21
- E. -10
- F. -1
- G. 10
- H. 21
- I. 28
- J. 36
- K. 40
- L. None of those above

$$\det(A^T) =$$

- A. -3
- B. -2
- C. -1
- D. 0
- E. 1
- F. 2
- G. 3
- H. 4
- I. None of those above

$$\det(B^{-1}) =$$

- A. -0.4
- B. -0.5
- C. 0
- D. 0.4
- E. 0.5
- F. 1
- G. None of those above

$$\det(B^4) =$$

- A. -81
- B. -36
- C. -12
- D. 0
- E. 12
- F. 36
- G. 81
- H. 16
- I. 1024
- J. None of those above

Correct Answers:

- B
- B
- E
- E
- H

50. (1 pt) local/Library/UI/Fall14/HW8.5.pg

Find the determinant of the matrix

$$A = \begin{bmatrix} -6 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ -8 & 1 & 3 & 0 \\ -6 & -3 & 1 & 3 \end{bmatrix}.$$

$$\det(A) =$$

- A. -400
- B. -360
- C. -288
- D. -120
- E. 0
- F. 120
- G. -108
- H. 240
- I. 360
- J. 400
- K. None of those above

Correct Answers:

- G

51. (1 pt) local/Library/UI/Fall14/HW8.7.pg

Suppose that a 4×4 matrix A with rows v_1, v_2, v_3 , and v_4 has determinant $\det A = -4$. Find the following determinants:

$$B = \begin{bmatrix} v_1 \\ 9v_2 \\ v_3 \\ v_4 \end{bmatrix} \quad \det(B) =$$

- A. -18
- B. -15
- C. -12
- D. -36
- E. -9
- F. 0
- G. 9
- H. 12
- I. 15
- J. 18
- K. None of those above

$$C = \begin{bmatrix} v_2 \\ v_1 \\ v_4 \\ v_3 \end{bmatrix} \quad \det(C) =$$

- A. -18

- B. -4
- C. -9
- D. -3
- E. 0
- F. 3
- G. 9
- H. 12
- I. 18
- J. None of those above

$$D = \begin{bmatrix} v_1 + 3v_3 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$

$\det(D) =$

- A. -18
- B. -4
- C. -9
- D. -3
- E. 0
- F. 3
- G. 9
- H. 12
- I. 18
- J. None of those above

Correct Answers:

- D
- B
- B

52. (1 pt) local/Library/UI/Fall14/HW8.8.pg

Use determinants to determine whether each of the following sets of vectors is linearly dependent or independent.

$$\begin{bmatrix} -9 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ -5 \end{bmatrix},$$

- A. Linearly Dependent
- B. Linearly Independent

$$\begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -18 \\ -1 \end{bmatrix}, \begin{bmatrix} 7 \\ 44 \\ -1 \end{bmatrix},$$

- A. Linearly Dependent
- B. Linearly Independent

$$\begin{bmatrix} -5 \\ -10 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix},$$

- A. Linearly Dependent
- B. Linearly Independent

$$\begin{bmatrix} 2 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} -4 \\ -8 \\ -8 \end{bmatrix}, \begin{bmatrix} 10 \\ 20 \\ 20 \end{bmatrix},$$

- A. Linearly Dependent
- B. Linearly Independent

Correct Answers:

- B
- B
- A
- A

53. (1 pt) local/Library/UI/Fall14/HW8.10.pg

$$A = \begin{bmatrix} -8 & 6 & 0 & 3 \\ -4 & -2 & 0 & 0 \\ 0 & -7 & 0 & 0 \\ 8 & -2 & -4 & 8 \end{bmatrix}$$

The determinant of the matrix is

- A. -1890
- B. -1024
- C. -630
- D. 336
- E. -210
- F. 0
- G. 324
- H. 630
- I. 1024
- J. None of those above

Hint: Find a good row or column and expand by minors.

Correct Answers:

- D

54. (1 pt) local/Library/UI/Fall14/HW8.11.pg

Find the determinant of the matrix

$$M = \begin{bmatrix} -3 & 0 & 0 & -2 & 0 \\ -3 & 0 & 3 & 0 & 0 \\ 0 & -3 & 0 & 0 & -2 \\ 0 & 0 & 0 & -3 & 2 \\ 0 & -2 & -2 & 0 & 0 \end{bmatrix}.$$

$\det(M) =$

- A. -48
- B. -35
- C. -20
- D. 180
- E. -5
- F. 5

- G. 18
- H. 20
- I. 81
- J. None of those above

Correct Answers:

- D

55. (1 pt) local/Library/UI/Fall14/HW8.12.pg

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

And now for the grand finale: The determinant of the matrix is

- A. -362880
- B. -5
- C. 0
- D. 5
- E. 20
- F. 30
- G. 40
- H. 362880
- I. None of the above

Hint: Remember that a square linear system has a unique solution if the determinant of the coefficient matrix is non-zero.

Solution: (Instructor solution preview: show the student solution after due date.)

Solution: Since all the rows are the same a linear system with A as its coefficient matrix cannot have a unique solution and therefore the determinant of A is zero.

Correct Answers:

- C

A system of equations can have exactly 2 solution.

- A. True
- B. False

Correct Answers:

- A

A system of linear equations can have exactly 2 solution.

- A. True
- B. False

Correct Answers:

- B

A system of linear equations has no solution if and only if the last column of its augmented matrix corresponds to a pivot column.

- A. True
- B. False

Correct Answers:

- A

A system of linear equations has an infinite number of solutions if and only if its associated augmented matrix has a column corresponding to a free variable.

- A. True
- B. False

Correct Answers:

- B

If a system of linear equations has an infinite number of solutions, then its associated augmented matrix has a column corresponding to a free variable.

- A. True
- B. False

Correct Answers:

- A

If a linear system has four equations and seven variables, then it must have infinitely many solutions.

- A. True
- B. False

Correct Answers:

- B

Every linear system with free variables has infinitely many solutions.

- A. True
- B. False

Correct Answers:

- B

Any linear system with more variables than equations cannot have a unique solution.

- A. True
- B. False

Correct Answers:

- A

If a linear system has the same number of equations and variables, then it must have a unique solution.

- A. True
- B. False

Correct Answers:

- B

A vector b is a linear combination of the columns of a matrix A if and only if the equation $Ax = b$ has at least one solution.

- A. True
- B. False

Correct Answers:

- A

If the columns of an $m \times n$ matrix, A span \mathbb{R}^m , then the equation, $Ax = b$ is consistent for each b in \mathbb{R}^m .

- A. True
- B. False

Correct Answers:

- A

If A is an $m \times n$ matrix and if the equation $Ax = b$ is inconsistent for some b in \mathbb{R}^m , then A cannot have a pivot position in every row.

- A. True
- B. False

Correct Answers:

- A

Any linear combination of vectors can always be written in the form Ax for a suitable matrix A and vector x .

- A. True
- B. False

Correct Answers:

- A

If the equation $Ax = b$ is inconsistent, then b is not in the set spanned by the columns of A .

- A. True
- B. False

Correct Answers:

- A

If A is an $m \times n$ matrix whose columns do not span \mathbb{R}^m , then the equation $Ax = b$ is inconsistent for some b in \mathbb{R}^m .

- A. True
- B. False

Correct Answers:

- A

Every linear system with free variables has infinitely many solutions.

- A. True
- B. False

Correct Answers:

- B

72. (1 pt) local/Library/UI/Fall14/quiz2.2.pg

Find the area of the triangle with vertices $(1, -2)$, $(8, -5)$, and $(3, 2)$.

Area =

- A. 2
- B. 5
- C. 6
- D. 8
- E. 9
- F. 12
- G. 17
- H. 20
- I. 25
- J. None of those above

Hint: The area of a triangle is half the area of a parallelogram. Find the vectors that determine the parallelogram of interest. If you have difficulty, visualizing the problem may be helpful: plot the 3 points.

Correct Answers:

- G

73. (1 pt) local/Library/UI/Fall14/quiz2.6.pg

Determine if v is an eigenvector of the matrix A .

1. $A = \begin{bmatrix} 8 & -4 & 15 \\ -7 & 5 & -15 \\ -6 & 4 & -13 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

- A. Yes
- B. No

2. $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -2 & 0 \\ 5 & -1 & -1 \end{bmatrix}$, $v = \begin{bmatrix} 6 \\ 6 \\ -1 \end{bmatrix}$

- A. Yes
- B. No

3. $A = \begin{bmatrix} -8 & -4 & -1 \\ 13 & 3 & -5 \\ -10 & -4 & 1 \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

- A. Yes
- B. No

4. $A = \begin{bmatrix} 6 & 5 & -16 \\ 6 & 5 & -16 \\ 3 & 5 & -13 \end{bmatrix}$, $v = \begin{bmatrix} 7 \\ 9 \\ 1 \end{bmatrix}$

- A. Yes
- B. No

Correct Answers:

- A
- B
- A
- B

74. (1 pt) local/Library/UI/Fall14/quiz2.7.pg

Given that $v_1 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$ are eigenvectors of the matrix $A = \begin{bmatrix} 62 & -168 \\ 21 & -57 \end{bmatrix}$, determine the corresponding eigenvalues.

a. $\lambda_1 =$

- A. -6
- B. -5
- C. -4
- D. -3
- E. -2
- F. -1
- G. 6
- H. 0
- I. 1
- J. 2
- K. None of those above

b. $\lambda_2 =$

- A. -5
- B. -4
- C. -3
- D. -2
- E. 0
- F. -1
- G. 1
- H. 2
- I. 3
- J. None of those above

Correct Answers:

- G
- F

75. (1 pt) local/Library/UI/Fall14/volume1.pg

Find the volume of the parallelepiped determined by vectors

$\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ -4 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} -2 \\ -2 \\ -5 \end{bmatrix}$

- A. 72
- B. -5
- C. -4
- D. -2

- E. -1
- F. 0
- G. 1
- H. 3
- I. 5
- J. 7
- K. None of those above

Correct Answers:

- A

Suppose a 3 x 5 augmented matrix contains a pivot in every row. Then the corresponding system of equations has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution, or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- E

Given the following augmented matrix,

$$A = \left[\begin{array}{ccccc} 2 & 0 & 3 & -2 & 4 \\ 0 & -5 & 4 & -6 & 4 \\ 0 & 0 & 0 & 0 & 5 \end{array} \right],$$

the corresponding system of equations has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- A

Given the following augmented matrix,

$$A = \left[\begin{array}{ccccc} -1 & -3 & 3 & -3 & 7 \\ 0 & -3 & 3 & 3 & 2 \\ 0 & 0 & 0 & -2 & -8 \end{array} \right],$$

the corresponding system of equations has

- A. No solution

- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- C

Suppose an augmented matrix contains a pivot in the last column. Then the corresponding system of equations has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- A

Suppose a coefficient matrix A contains a pivot in the last column. Then $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- G

Suppose a coefficient matrix A contains a pivot in every row. Then $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- F

Suppose a coefficient matrix A contains a pivot in every column. Then $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- D

Suppose $A\vec{x} = \vec{0}$ has an infinite number of solutions, then given a vector \vec{b} of the appropriate dimension, $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- E

Suppose $A\vec{x} = \vec{b}$ has an infinite number of solutions, then $A\vec{x} = \vec{0}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- C

Suppose $A\vec{x} = \vec{0}$ has a unique solution, then given a vector \vec{b} of the appropriate dimension, $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- D

Suppose $A\vec{x} = \vec{b}$ has a unique solution, then $A\vec{x} = \vec{0}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- B

Suppose $A\vec{x} = \vec{b}$ has no solution, then $A\vec{x} = \vec{0}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution

- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- F

Suppose A is a square matrix and $A\vec{x} = \vec{0}$ has a unique solution, then given a vector \vec{b} of the appropriate dimension, $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- B

Suppose A is a square matrix and $A\vec{x} = \vec{0}$ has an infinite number of solutions, then given a vector \vec{b} of the appropriate dimension, $A\vec{x} = \vec{b}$ has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Correct Answers:

- E

90. (1 pt) local/Library/UI/LinearSystems/spanHW4.pg

$$\text{Let } A = \begin{bmatrix} -5 & 5 & 0 \\ 5 & -7 & 4 \\ -3 & 5 & -3 \end{bmatrix}, \text{ and } b = \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}.$$

Denote the columns of A by a_1, a_2, a_3 , and let $W = \text{span}\{a_1, a_2, a_3\}$.

- ☐ 1. Determine if b is in $\{a_1, a_2, a_3\}$
- ☐ 2. Determine if b is in W

How many vectors are in $\{a_1, a_2, a_3\}$? (For infinitely many, enter -1) _____

How many vectors are in W ? (For infinitely many, enter -1) _____

Correct Answers:

- No
- Yes
- 3
- -1

91. (1 pt) local/Library/UI/MatrixAlgebra/Euclidean/2.3.43.pg

Let A be a matrix with linearly independent columns. Select the best statement.

- A. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it has more rows than columns.
- B. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it has more columns than rows.
- C. The equation $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions precisely when it is a square matrix.
- D. The equation $A\mathbf{x} = \mathbf{0}$ always has nontrivial solutions.
- E. There is insufficient information to determine if such an equation has nontrivial solutions.
- F. The equation $A\mathbf{x} = \mathbf{0}$ never has nontrivial solutions.
- G. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

The linear independence of the columns does not change with row reduction. Since the columns are linearly independent, after row reduction, each column contains a leading 1. We get nontrivial solutions when we have columns without a leading 1 in the row reduced matrix.

The equation $A\mathbf{x} = \mathbf{0}$ never has nontrivial solutions.

Correct Answers:

- F

92. (1 pt) local/Library/UI/MatrixAlgebra/Euclidean/2.3.44.pg

Let A be a matrix with linearly independent columns. Select the best statement.

- A. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} precisely when it has more rows than columns.
- B. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} precisely when it has more columns than rows.
- C. There is insufficient information to determine if $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} .
- D. The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} precisely when it is a square matrix.
- E. The equation $A\mathbf{x} = \mathbf{b}$ never has a solution for all \mathbf{b} .

- F. The equation $A\mathbf{x} = \mathbf{b}$ always has a solution for all \mathbf{b} .
- G. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

A linear equation has a solution when the row reduced form of the augmented solution does not have a leading 1 in the extra column that corresponds to constants. Since the columns of the matrix are linearly independent the number of columns is no more than the number of rows. If there are fewer columns than rows we can produce a \mathbf{b} for which there is no solution.

The equation $A\mathbf{x} = \mathbf{b}$ has a solution for all \mathbf{b} precisely when it is a square matrix.

Correct Answers:

- D

93. (1 pt) local/Library/UI/problem7.pg

A and B are $n \times n$ matrices.

Adding a multiple of one row to another does not affect the determinant of a matrix.

- A. True
- B. False

If the columns of A are linearly dependent, then $\det A = 0$.

- A. True
- B. False

$$\det(A+B) = \det A + \det B.$$

- A. True
- B. False

Correct Answers:

- A
- A
- B

Suppose A is a 8×6 matrix. If rank of $A = 2$, then nullity of $A =$

- A. -4
- B. -3

- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- I

The vector \vec{b} is NOT in $ColA$ if and only if $A\vec{v} = \vec{b}$ does NOT have a solution

- A. True
- B. False

Correct Answers:

- A

The vector \vec{b} is in $ColA$ if and only if $A\vec{v} = \vec{b}$ has a solution

- A. True
- B. False

Correct Answers:

- A

The vector \vec{v} is in $NullA$ if and only if $A\vec{v} = \vec{0}$

- A. True
- B. False

Correct Answers:

- A

If the equation $A\vec{x} = \vec{b}_1$ has at least one solution and if the equation $A\vec{x} = \vec{b}_2$ has at least one solution, then the equation $A\vec{x} = 8\vec{b}_1 - 7\vec{b}_2$ also has at least one solution.

- A. True
- B. False

Hint: (Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)

Is $colA$ a subspace? Is $colA$ closed under linear combinations?

Correct Answers:

- A

If \vec{x}_1 and \vec{x}_2 are solutions to $A\vec{x} = \vec{0}$, then $-5\vec{x}_1 - 8\vec{x}_2$ is also a solution to $A\vec{x} = \vec{0}$.

- A. True
- B. False

Hint: (Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)

Is $Nul A$ a subspace? Is $Nul A$ closed under linear combinations?

Correct Answers:

- A

If \vec{x}_1 and \vec{x}_2 are solutions to $A\vec{x} = \vec{b}$, then $-1\vec{x}_1 + 1\vec{x}_2$ is also a solution to $A\vec{x} = \vec{b}$.

- A. True
- B. False

Hint: (Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)

Is the solution set to $A\vec{x} = \vec{b}$ a subspace even when \vec{b} is not $\vec{0}$? Is the solution set to $A\vec{x} = \vec{b}$ closed under linear combinations even when \vec{b} is not $\vec{0}$?

Correct Answers:

- B

Find the area of the parallelogram determined by the vectors $\begin{bmatrix} 2.375 \\ 4 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 8 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

Correct Answers:

- F

Suppose A is a 8×4 matrix. Then $nul A$ is a subspace of R^k where $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2

- H. 3
- I. 4
- J. none of the above

Correct Answers:

- I

Suppose A is a 3×7 matrix. Then $col A$ is a subspace of R^k where $k =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- H

Calculate the determinant of $\begin{bmatrix} 10.3333333333333 & 7 \\ 5 & 3 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

Correct Answers:

- A

Suppose $A \begin{bmatrix} -5 \\ -3 \\ -4 \end{bmatrix} = \begin{bmatrix} 20 \\ 12 \\ 16 \end{bmatrix}$. Then an eigenvalue of A is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- A

106. (1 pt) local/Library/UI/volumn2.pg

A and B are $n \times n$ matrices.

Check the true statements below:

- A. If A is 3×3 , with columns a_1, a_2, a_3 , then $\det A$ equals the volume of the parallelepiped determined by the vectors a_1, a_2, a_3 .
- B. If A is 3×3 , with columns a_1, a_2, a_3 , then the absolute value of $\det A$ equals the volume of the parallelepiped determined by the vectors a_1, a_2, a_3 .
- C. $\det A^T = (-1)\det A$.

Correct Answers:

- B

107. (1 pt) UI/DIAGtfproblem1.pg

A, P and D are $n \times n$ matrices.

Check the true statements below:

- A. A is diagonalizable if A has n distinct linearly independent eigenvectors.
- B. If $AP = PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A .
- C. A is diagonalizable if $A = PDP^{-1}$ for some diagonal matrix D and some invertible matrix P .
- D. A is diagonalizable if and only if A has n eigenvalues, counting multiplicities.
- E. If there exists a basis for \mathbb{R}^n consisting entirely of eigenvectors of A , then A is diagonalizable.
- F. A is diagonalizable if A has n distinct eigenvectors.
- G. If A is diagonalizable, then A has n distinct eigenvalues.
- H. If A is symmetric, then A is orthogonally diagonalizable.
- I. If A is orthogonally diagonalizable, then A is symmetric.
- J. If A is diagonalizable, then A is invertible.
- K. If A is symmetric, then A is diagonalizable.
- L. If A is invertible, then A is diagonalizable.
- M. If A is diagonalizable, then A is symmetric.

Correct Answers:

- ABCEHIK

108. (1 pt) UI/orthog.pg

All vectors and subspaces are in \mathbb{R}^n .

Check the true statements below:

- A. If $A\mathbf{v} = r\mathbf{v}$ and $A\mathbf{w} = s\mathbf{w}$ and $r \neq s$, then $\mathbf{v} \cdot \mathbf{w} = 0$.
- B. If $W = \text{Span}\{x_1, x_2, x_3\}$ and if $\{v_1, v_2, v_3\}$ is an orthonormal set in W , then $\{v_1, v_2, v_3\}$ is an orthonormal basis for W .

- C. In a QR factorization, say $A = QR$ (when A has linearly independent columns), the columns of Q form an orthonormal basis for the column space of A .
- D. If x is not in a subspace W , then $x - \text{proj}_W(x)$ is not zero.
- E. If $\{v_1, v_2, v_3\}$ is an orthonormal set, then the set $\{v_1, v_2, v_3\}$ is linearly independent.
- F. If \mathbf{v} and \mathbf{w} are both eigenvectors of A and if A is symmetric, then $\mathbf{v} \cdot \mathbf{w} = 0$.
- G. If A is symmetric, $A\mathbf{v} = r\mathbf{v}$, $A\mathbf{w} = s\mathbf{w}$ and $r \neq s$, then $\mathbf{v} \cdot \mathbf{w} = 0$.

Correct Answers:

- BCDEG

109. (1 pt) local/Library/UI/eigenTF.pg

A is $n \times n$ an matrices.

Check the true statements below:

- A. The vector $\mathbf{0}$ can never be an eigenvector of A .
- B. 0 can never be an eigenvalue of A .
- C. The eigenspace corresponding to a particular eigenvalue of A contains an infinite number of vectors.
- D. 0 is an eigenvalue of A if and only if the columns of A are linearly dependent.
- E. A will have at most n eigenvectors.
- F. The vector $\mathbf{0}$ is an eigenvector of A if and only if $Ax = 0$ has a nonzero solution.
- G. There are an infinite number of eigenvectors that correspond to a particular eigenvalue of A .
- H. 0 is an eigenvalue of A if and only if $\det(A) = 0$.
- I. A will have at most n eigenvalues.
- J. 0 is an eigenvalue of A if and only if $Ax = 0$ has a nonzero solution.
- K. The vector $\mathbf{0}$ is an eigenvector of A if and only if the columns of A are linearly dependent.
- L. The vector $\mathbf{0}$ is an eigenvector of A if and only if $\det(A) = 0$.
- M. 0 is an eigenvalue of A if and only if $Ax = 0$ has an infinite number of solutions.

Correct Answers:

- ACDGHIJM

Suppose $A = PDP^{-1}$ where D is a diagonal matrix. If $P = [\vec{p}_1 \ \vec{p}_2 \ \vec{p}_3]$, then $2\vec{p}_1$ is an eigenvector of A

- A. True
- B. False

Correct Answers:

- A

Suppose $A = PDP^{-1}$ where D is a diagonal matrix. If $P = [\vec{p}_1 \ \vec{p}_2 \ \vec{p}_3]$, then $\vec{p}_1 + \vec{p}_2$ is an eigenvector of A

- A. True
- B. False

Correct Answers:

- B

Suppose $A = PDP^{-1}$ where D is a diagonal matrix. Suppose also the d_{ii} are the diagonal entries of D . If $P = [\vec{p}_1 \ \vec{p}_2 \ \vec{p}_3]$ and $d_{11} = d_{22}$, then $\vec{p}_1 + \vec{p}_2$ is an eigenvector of A

- A. True
- B. False

Correct Answers:

- A

Suppose $A = PDP^{-1}$ where D is a diagonal matrix. Suppose also the d_{ii} are the diagonal entries of D . If $P = [\vec{p}_1 \ \vec{p}_2 \ \vec{p}_3]$ and $d_{22} = d_{33}$, then $\vec{p}_1 + \vec{p}_2$ is an eigenvector of A

- A. True
- B. False

Correct Answers:

- B

If \vec{v}_1 and \vec{v}_2 are eigenvectors of A corresponding to eigenvalue λ_0 , then $1\vec{v}_1 + 9\vec{v}_2$ is also an eigenvector of A corresponding to eigenvalue λ_0 when $1\vec{v}_1 + 9\vec{v}_2$ is not $\vec{0}$.

- A. True
- B. False

Hint: (Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)

Is an eigenspace a subspace? Is an eigenspace closed under linear combinations?

Also, is $1\vec{v}_1 + 9\vec{v}_2$ nonzero?

Correct Answers:

- A

Which of the following is an eigenvalue of $\begin{bmatrix} -8 & -20 \\ 1 & 4 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- G

Let $A = \begin{bmatrix} 9 & -1 & 1 \\ 0 & -8 & -4 \\ 0 & 0 & 9 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

Note that since the matrix is triangular, the eigenvalues are the diagonal elements 9 and -8. Since A is a 3 x 3 matrix, we need 3 linearly independent eigenvectors. Since -8 has algebraic multiplicity 1, it has geometric multiplicity 1 (the dimension of its eigenspace is 1). Thus we can only use one eigenvector from the eigenspace corresponding to eigenvalue -8 to form P .

Thus we need 2 linearly independent eigenvectors from the eigenspace corresponding to the other eigenvalue 9. The eigenvalue 9 has algebraic multiplicity 2. Let E = dimension of the eigenspace corresponding eigenvalue 9. Then $1 \leq E \leq 2$. But we can easily see that the Nullspace of $A - 9I$ has dimension 1.

Thus we do not have enough linearly independent eigenvectors to form P . Hence A is not diagonalizable.

Correct Answers:

- B

Let $A = \begin{bmatrix} -6 & -25 & 40 \\ 0 & -1 & -8 \\ 0 & 0 & -6 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

Note that since the matrix is triangular, the eigenvalues are the diagonal elements -6 and -1. Since A is a 3 x 3 matrix, we need 3 linearly independent eigenvectors. Since -1 has algebraic multiplicity 1, it has geometric multiplicity 1 (the dimension of its eigenspace is 1). Thus we can only use one eigenvector from the eigenspace corresponding to eigenvalue -1 to form P .

Thus we need 2 linearly independent eigenvectors from the eigenspace corresponding to the other eigenvalue -6. The eigenvalue -6 has algebraic multiplicity 2. Let $E =$ dimension of the eigenspace corresponding eigenvalue -6. Then $1 \leq E \leq 2$. But we can easily see that the Nullspace of $A + 6I$ has dimension 2.

Thus we have 3 linearly independent eigenvectors which we can use to form the square matrix P . Hence A is diagonalizable.

Correct Answers:

- A

Let $A = \begin{bmatrix} 1 & 9 \\ 8 & -8 \end{bmatrix}$. Is A = diagonalizable?

- A. yes
- B. no
- C. none of the above

Hint: (Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)

You do NOT need to do much work for this problem. You just need to know if the matrix A is diagonalizable. Since A is a 2×2 matrix, you need 2 linearly independent eigenvectors of A to form P . Does A have 2 linearly independent eigenvectors? Note you don't need to know what these eigenvectors are. You don't even need to know the eigenvalues.

Solution: (Instructor solution preview: show the student solution after due date.)

SOLUTION

The matrix A has 2 eigenvalues (note you do not need to know their values, just that you have 2 distinct eigenvalues). Hence the A is diagonalizable by the following: Each eigenvalue has a 1-dimensional eigenspace. If one takes one eigenvector (any nonzero element of the eigenspace) from each eigenspace, then that pair forms a linearly independent set and can be used to form P . Since A is a 2×2 matrix, one only needs 2 linearly independent eigenvectors to form P .

Sidenote 1: This works whenever one has n distinct eigenvalues for an $n \times n$ matrix. The only time a matrix is not diagonalizable is when there exists an eigenvalue whose geometric multiplicity is strictly less than its algebraic multiplicity. Then you will not have enough linearly independent eigenvectors to form P .

Sidenote 2: This works even if the eigenvalues are complex number and not real numbers, but we won't handle that case in this class (but the algorithm is identical to that for real eigenvalues).

Correct Answers:

- A

Let $A = \begin{bmatrix} -0.41666666666667 & -9.6666666666667 & 5.4166666666667 \\ -2.0416666666667 & -6.1666666666667 & -3.2916666666667 \\ -0.58333333333333 & -2.3333333333333 & -7.7500000000000 \end{bmatrix}$
and let $P = \begin{bmatrix} -8 & 3 & -4 \\ 2 & 7 & 3 \\ 0 & 7 & 8 \end{bmatrix}$.

Suppose $A = PDP^{-1}$. Then if d_{ii} are the diagonal entries of D , $d_{11} =$,

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

Hint: (Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)

Use definition of eigenvalue since you know an eigenvector corresponding to eigenvalue d_{11} .

Correct Answers:

- G

Calculate the dot product: $\begin{bmatrix} -5 \\ -1 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ -4 \\ 2.6666666666667 \end{bmatrix}$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- F

Determine the length of $\begin{bmatrix} 2 \\ 3.46410161513775 \end{bmatrix}$.

- A. -4
- B. -3
- C. -2
- D. -1

- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

Correct Answers:

- I

If the characteristic polynomial of $A = (\lambda - 6)^5(\lambda - 8)(\lambda + 9)^8$, then the algebraic multiplicity of $\lambda = 8$ is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- F

If the characteristic polynomial of $A = (\lambda - 7)^7(\lambda - 3)(\lambda + 8)^3$, then the geometric multiplicity of $\lambda = 3$ is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- F

If the characteristic polynomial of $A = (\lambda + 2)^5(\lambda + 5)^2(\lambda - 6)^5$, then the algebraic multiplicity of $\lambda = -5$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2

- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

Correct Answers:

- C

If the characteristic polynomial of $A = (\lambda + 1)^8(\lambda + 4)^2(\lambda + 2)^8$, then the geometric multiplicity of $\lambda = -4$ is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

Correct Answers:

- G

Suppose the orthogonal projection of $\begin{bmatrix} -224 \\ -1 \\ -3 \end{bmatrix}$ onto

$\begin{bmatrix} 1 \\ -5 \\ -5 \end{bmatrix}$ is (z_1, z_2, z_3) . Then $z_1 =$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- A

Suppose $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$ is a unit vector in the direction of

$\begin{bmatrix} -4 \\ 4 \\ 3.52766841475279 \end{bmatrix}$. Then $u_1 =$

- A. -0.8
- B. -0.6
- C. -0.4
- D. -0.2

- E. 0
- F. 0.2
- G. 0.4
- H. 0.6
- I. 0.8
- J. 1

Correct Answers:

- B

128. (1 pt) Library/Rochester/setLinearAlgebra23QuadraticForms-ur.Ja.23.2.pg

Find the eigenvalues of the matrix

$$M = \begin{bmatrix} 30 & 50 \\ 50 & 30 \end{bmatrix}.$$

Enter the two eigenvalues, separated by a comma:

Classify the quadratic form $Q(x) = x^T A x$:

- A. $Q(x)$ is positive semidefinite
- B. $Q(x)$ is indefinite
- C. $Q(x)$ is positive definite
- D. $Q(x)$ is negative semidefinite
- E. $Q(x)$ is negative definite

Correct Answers:

- 80, -20
- B

129. (1 pt) Library/Rochester/setLinearAlgebra23QuadraticForms-ur.Ja.23.3.pg

The matrix

$$A = \begin{bmatrix} -6 & 0 & 0 \\ 0 & -1.4 & -0.8 \\ 0 & -0.8 & -2.6 \end{bmatrix}$$

has three distinct eigenvalues, $\lambda_1 < \lambda_2 < \lambda_3$,

$\lambda_1 =$ __,

$\lambda_2 =$ __,

$\lambda_3 =$ __.

Classify the quadratic form $Q(x) = x^T A x$:

- A. $Q(x)$ is positive semidefinite
- B. $Q(x)$ is negative semidefinite
- C. $Q(x)$ is negative definite
- D. $Q(x)$ is indefinite
- E. $Q(x)$ is positive definite

Correct Answers:

- -6
- -3
- -1
- C

Use Cramer's rule to solve the following system of equations for x :

$$\begin{aligned} -22x + 6y &= 8 \\ -4x + 1y &= 0 \end{aligned}$$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Correct Answers:

- I

131. (1 pt) local/Library/UI/Fall14/quiz2.9.pg

Suppose A is an invertible $n \times n$ matrix and v is an eigenvector of A with associated eigenvalue 6. Convince yourself that v is an eigenvector of the following matrices, and find the associated eigenvalues:

1. A^5 , eigenvalue =

- A. 16
- B. 81
- C. 125
- D. 216
- E. 1024
- F. 7776
- G. 2000
- H. None of those above

2. A^{-1} , eigenvalue =

- A. -0.5
- B. -0.333
- C. 0.166666666666667
- D. -0.125
- E. 0
- F. 0.125
- G. 0.333
- H. 0.5
- I. None of those above

3. $A + 6I_n$, eigenvalue =

- A. -8
- B. -4
- C. -5

- D. 0
- E. 2
- F. 4
- G. 12
- H. 10
- I. None of those above

4. 9A, eigenvalue =

- A. -40
- B. -36
- C. -28
- D. 54
- E. -12
- F. 0
- G. 24
- H. 36
- I. None of those above

Correct Answers:

- F
- C
- G
- D

132. (1 pt) local/Library/UI/Fall14/quiz2.10.pg

If $v_1 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$

are eigenvectors of a matrix A corresponding to the eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -2$, respectively, then

a. $A(v_1 + v_2) =$

- A. $\begin{bmatrix} -3 \\ 5 \end{bmatrix}$
- B. $\begin{bmatrix} -2 \\ 4 \end{bmatrix}$
- C. $\begin{bmatrix} -6 \\ 5 \end{bmatrix}$
- D. $\begin{bmatrix} 10 \\ 6 \end{bmatrix}$
- E. $\begin{bmatrix} 12 \\ 4 \end{bmatrix}$
- F. $\begin{bmatrix} 0 \\ 12 \end{bmatrix}$
- G. None of those above

b. $A(-3v_1) =$

- A. $\begin{bmatrix} -12 \\ -12 \end{bmatrix}$
- B. $\begin{bmatrix} -2 \\ 8 \end{bmatrix}$

- C. $\begin{bmatrix} -6 \\ 4 \end{bmatrix}$
- D. $\begin{bmatrix} 10 \\ 6 \end{bmatrix}$
- E. $\begin{bmatrix} 6 \\ -6 \end{bmatrix}$
- F. $\begin{bmatrix} 12 \\ 4 \end{bmatrix}$
- G. $\begin{bmatrix} 0 \\ 12 \end{bmatrix}$
- H. None of those above

Correct Answers:

- F
- E

133. (1 pt) local/Library/UI/Fall14/quiz2.11.pg

Let $v_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

be eigenvectors of the matrix A which correspond to the eigenvalues $\lambda_1 = -3$, $\lambda_2 = 1$, and $\lambda_3 = 2$, respectively, and let

$$v = \begin{bmatrix} -5 \\ -1 \\ 0 \end{bmatrix}.$$

Express v as a linear combination of v_1 , v_2 , and v_3 , and find Av .

1. If $v = c_1v_1 + c_2v_2 + c_3v_3$, then $(c_1, c_2, c_3) =$

- A. (1,2,2)
- B. (-3,2,4)
- C. (-4,7,3)
- D. (-2,-1,-2)
- E. (0,1,2)
- F. (4,-1,5)
- G. None of above

2. $Av =$

- A. $\begin{bmatrix} -12 \\ 7 \\ -12 \end{bmatrix}$
- B. $\begin{bmatrix} -2 \\ 12 \\ 8 \end{bmatrix}$
- C. $\begin{bmatrix} -6 \\ 7 \\ 4 \end{bmatrix}$
- D. $\begin{bmatrix} 10 \\ 0 \\ 6 \end{bmatrix}$

- E. $\begin{bmatrix} -7 \\ 7 \\ 10 \end{bmatrix}$
- F. $\begin{bmatrix} 12 \\ 8 \\ 4 \end{bmatrix}$
- G. $\begin{bmatrix} -7 \\ -3 \\ 12 \end{bmatrix}$
- H. None of those above

Correct Answers:

- D
- E

Suppose u and v are eigenvectors of A with eigenvalue 1 and w is an eigenvector of A with eigenvalue 2. Determine which of the following are eigenvectors of A and their corresponding eigenvalues.

(a.) If $4v$ an eigenvector of A , determine its eigenvalue. Else state it is not an eigenvector of A .

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. $4v$ need not be an eigenvector of A

(b.) If $2u + 3v$ an eigenvector of A , determine its eigenvalue. Else state it is not an eigenvector of A .

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. $2u + 3v$ need not be an eigenvector of A

(c.) If $2u + 3w$ an eigenvector of A , determine its eigenvalue. Else state it is not an eigenvector of A .

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. $2u + 3w$ need not be an eigenvector of A

Correct Answers:

- F
- F
- J