### Assignment FinalExamReviewMultChoiceLongForm due 12/31/2014 at 04:54pm CST

# $\begin{array}{lll} \textbf{1.} & (1 & pt) & Library/Rochester/setLinearAlgebra4InverseMatrix-/ur\_Ch2\_1\_4.pg \end{array}$

Are the following matrices invertible? Enter "Y" or "N". You must get all of the answers correct to receive credit.

$$\begin{array}{ccccc}
 & -1 & \begin{bmatrix} -4 & 0 \\ 0 & 0 \end{bmatrix} \\
 & -2 & \begin{bmatrix} 7 & 0 \\ 0 & -6 \end{bmatrix} \\
 & -3 & \begin{bmatrix} 2 & 0 \\ -4 & 0 \end{bmatrix} \\
 & -4 & \begin{bmatrix} -2 & 4 \\ -4 & -5 \end{bmatrix}
\end{array}$$

#### 

The matrix  $\begin{bmatrix} 2 & -6 \\ -5 & k \end{bmatrix}$  is invertible if and only if  $k \neq$ \_\_\_\_.

## ${\bf 3.} \qquad (1 \quad pt) \quad Library/Rochester/setLinearAlgebra 9 Dependence-/ur\_la\_9\_7.pg$

The vectors

The vectors
$$v = \begin{bmatrix} -4 \\ -9 \\ -5 \end{bmatrix}, u = \begin{bmatrix} -3 \\ -9 \\ -12 + k \end{bmatrix}, \text{ and } w = \begin{bmatrix} 3 \\ 11 \\ 8 \end{bmatrix}.$$
are linearly independent if and only if  $k \neq -\infty$ .

#### 

Express the vector  $v = \begin{bmatrix} 14 \\ -26 \end{bmatrix}$  as a linear combination of  $x = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$  and  $y = \begin{bmatrix} -5 \\ 2 \end{bmatrix}$ .

## ${\bf 5.} \qquad (1 \quad pt) \quad Library/TCNJ/TCNJ\_BasesLinearlyIndependentSet-/problem5.pg$

Let  $W_1$  be the set:  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ .

Determine if  $W_1$  is a basis for  $\mathbb{R}^3$  and check the correct answer(s) below.

- A.  $W_1$  is not a basis because it does not span  $\mathbb{R}^3$ .
- B.  $W_1$  is a basis.
- C.  $W_1$  is not a basis because it is linearly dependent.

Let 
$$W_2$$
 be the set:  $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ .

Determine if  $W_2$  is a basis for  $\mathbb{R}^3$  and check the correct answer(s) below.

- A. W<sub>2</sub> is not a basis because it is linearly dependent.
- B.  $W_2$  is not a basis because it does not span  $\mathbb{R}^3$ .
- C.  $W_2$  is a basis.

#### 6. (1 pt) Library/TCNJ/TCNJ\_LinearIndependence/problem3.pg

If k is a real number, then the vectors (1,k),(k,k+56) are linearly independent precisely when

$$k \neq a, b$$
, where  $a = \underline{\hspace{1cm}}, b = \underline{\hspace{1cm}}$ , and  $a < b$ .

### 7. (1 pt) Library/TCNJ/TCNJ\_LinearSystems/problem1.pg

Determine whether the following system has no solution, an infinite number of solutions or a unique solution.

$$\begin{array}{rclrcl}
 7x & -7y & = & 7 \\
 2x & -7y & = & 2 \\
 -11x & +21y & = & -11 \\
 7x & -7y & = & 7 \\
 \hline
 ?2. & 2x & -7y & = & 2 \\
 & -11x & +21y & = & -13 \\
 & 16x & +12y & = & -4 \\
 \hline
 ?3. & 12x & +9y & = & -3 \\
 & -28x & -21y & = & 7
\end{array}$$

#### 8. (1 pt) Library/TCNJ/TCNJ\_LinearSystems/problem2.pg

Determine whether the following system has no solution, an infinite number of solutions or a unique solution.

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#### 9. (1 pt) Library/TCNJ/TCNJ\_LinearSystems/problem3.pg

Give a geometric description of the following systems of equations

$$\begin{array}{rcl}
-20x + 25y & = & -5 \\
-8x + 10y & = & -2 \\
24x - 30y & = & 6 \\
3x - 5y & = & 7 \\
\hline
?2. & 6x + 3y & = & 3 \\
-3x - 21y & = & 16 \\
3x - 5y & = & 7 \\
\hline
?3. & 6x + 3y & = & 3 \\
-3x - 21y & = & 15
\end{array}$$

#### 10. (1 pt) Library/TCNJ/TCNJ\_LinearSystems/problem4.pg

Give a geometric description of the following system of equations

#### 11. (1 pt) Library/TCNJ/TCNJ\_LinearSystems/problem11.pg

Give a geometric description of the following systems of equations.

$$\begin{array}{rcrcr}
? 1. & -6x & -6y & = 0 \\
15x & +15y & = 1 \\
? 2. & -6x & -6y & = 0 \\
15x & +15y & = 0 \\
? 3. & -3x & +9y & = 3 \\
4x & -6y & = -5
\end{array}$$

#### ${\bf 12.}\ (1\ pt)\ Library/TCNJ/TCNJ\_MatrixEquations/problem 4.pg$

Let 
$$A = \begin{bmatrix} -5 & 1 & 3 \\ 3 & -3 & 2 \\ 3 & -1 & 3 \end{bmatrix}$$
 and  $x = \begin{bmatrix} -4 \\ -5 \\ 1 \end{bmatrix}$ .

|?|1. What does Ax mean?

#### 13. (1 pt) Library/TCNJ/TCNJ\_MatrixEquations/problem13.pg Do the following sets of vectors span $\mathbb{R}^3$ ?

$$\begin{array}{c}
? 3. \begin{bmatrix} -1 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix} \\
? 4. \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ 6 \\ 4 \end{bmatrix}$$

#### 14. (1 pt) Library/TCNJ/TCNJ\_VectorEquations/problem5.pg

Let  $H = span\{u, v\}$ . For each of the following sets of vectors determine whether H is a line or a plane.

$$\begin{array}{c}
? 1. \ u = \begin{bmatrix} -1 \\ -3 \\ -2 \end{bmatrix}, v = \begin{bmatrix} 2 \\ 8 \\ 6 \end{bmatrix}, \\
? 2. \ u = \begin{bmatrix} -4 \\ -5 \\ 5 \end{bmatrix}, v = \begin{bmatrix} 15 \\ 20 \\ -22 \end{bmatrix}, \\
? 3. \ u = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}, v = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \\
? 4. \ u = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}, v = \begin{bmatrix} 6 \\ -4 \\ 2 \end{bmatrix},$$

#### 15. (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/2.2.8.pg

Let 
$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$
 and  $\mathbf{b} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$ .  
Is  $\mathbf{b}$  in the span of of  $\mathbf{a}_1$ ?

- A. Yes, **b** is in the span.
- B. No, **b** is not in the span.
- C. We cannot tell if **b** is in the span.

Either fill in the coefficients of the vector equation, or enter "NONE" if no solution is possible.

$$b = _{\_\_} a_1$$

#### **16.** (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/2.2.31.pg

$$Let A = \begin{bmatrix} 3 & 6 \\ -4 & 17 \\ -7 & 5 \end{bmatrix}.$$

We want to determine if the system  $A\mathbf{x} = \mathbf{b}$  has a solution for every  $\mathbf{b} \in \mathbb{R}^3$ .

Select the best answer.

- A. There is not a solution for every  $\mathbf{b} \in \mathbb{R}^3$  since 2 < 3.
- B. There is a solution for every  $\mathbf{b} \in \mathbb{R}^3$  since 2 < 3
- C. There is a solution for every  $\mathbf{b} \in \mathbb{R}^3$  but we need to row reduce A to show this.
- D. There is a not solution for every  $\mathbf{b} \in \mathbb{R}^3$  but we need to row reduce A to show this.
- E. We cannot tell if there is a solution for every  $\mathbf{b} \in \mathbb{R}^3$ .

## $17. \qquad (1\ pt)\ Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/2.2.56.pg$

What conditions on a matrix A insures that  $A\mathbf{x} = \mathbf{b}$  has a solution for all  $\mathbf{b}$  in  $\mathbb{R}^n$ ?

Select the best statement. (The best condition should work with any positive integer n.)

- A. The equation will have a solution for all **b** in  $\mathbb{R}^n$  as long as no column of *A* is a scalar multiple of another column.
- B. The equation will have a solution for all b in ℝ<sup>n</sup> as long as the columns of A do not include the zero column.
- C. There is no easy test to determine if the equation will have a solution for all **b** in  $\mathbb{R}^n$ .
- D. The equation will have a solution for all **b** in  $\mathbb{R}^n$  as long as the columns of A span  $\mathbb{R}^n$ .
- E. none of the above

### ${\bf 18.} \qquad (1\ pt)\ Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/2.2.57.pg$

Assume  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  spans  $\mathbb{R}^3$ . Select the best statement.

- A.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  spans  $\mathbb{R}^3$  unless  $\mathbf{u}_4$  is the zero vector.
- B.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  always spans  $\mathbb{R}^3$ .
- C.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  spans  $\mathbb{R}^3$  unless  $\mathbf{u}_4$  is a scalar multiple of another vector in the set.
- D. There is no easy way to determine if {u<sub>1</sub>, u<sub>2</sub>, u<sub>3</sub>, u<sub>4</sub>} spans R<sup>3</sup>.
- E.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  never spans  $\mathbb{R}^3$ .
- F. none of the above

### 19. (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/2.2.58.pg

Assume  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  does not span  $\mathbb{R}^3$ . Select the best statement.

- A.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  never spans  $\mathbb{R}^3$ .
- B.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  spans  $\mathbb{R}^3$  unless  $\mathbf{u}_4$  is a scalar multiple of another vector in the set.
- C.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  spans  $\mathbb{R}^3$  unless  $\mathbf{u}_4$  is the zero vector.
- D.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  may, but does not have to, span  $\mathbb{R}^3$ .
- E.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  always spans  $\mathbb{R}^3$ .
- F. none of the above

# ${\bf 20.} \qquad (1\ \ pt)\ \ Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/2.3.40.pg$

Let **S** be a set of *m* vectors in  $\mathbb{R}^n$  with m > n.

Select the best statement.

- A. The set **S** is linearly independent.
- B. The set S is linearly independent, as long as it does not include the zero vector.
- C. The set **S** is linearly dependent.
- D. The set **S** could be either linearly dependent or linearly independent, depending on the case.
- E. The set **S** is linearly independent, as long as no vector in **S** is a scalar multiple of another vector in the set.
- F. none of the above

## ${\bf 21.} \qquad (1\ pt)\ Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/2.3.41.pg$

Let *A* be a matrix with more rows than columns. Select the best statement.

- A. The columns of *A* must be linearly dependent.
- B. The columns of A are linearly independent, as long as no column is a scalar multiple of another column in A
- C. The columns of A are linearly independent, as long as they does not include the zero vector.
- D. The columns of *A* could be either linearly dependent or linearly independent depending on the case.
- E. The columns of A must be linearly independent.
- F. none of the above

### ${\bf 22.} \qquad (1\ \ pt)\ \ Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/2.3.42.pg$

Let *A* be a matrix with more columns than rows. Select the best statement.

- A. The columns of *A* are linearly independent, as long as they does not include the zero vector.
- B. The columns of A are linearly independent, as long as no column is a scalar multiple of another column in A
- C. The columns of *A* could be either linearly dependent or linearly independent depending on the case.
- D. The columns of *A* must be linearly dependent.
- E. none of the above

# ${\bf 23.} \qquad (1\ \ pt)\ \ Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/2.3.46.pg$

Let  $\{u_1, u_2, u_3\}$  be a linearly dependent set of vectors. Select the best statement.

• A. {**u**<sub>1</sub>, **u**<sub>2</sub>, **u**<sub>3</sub>, **u**<sub>4</sub>} is a linearly independent set of vectors unless **u**<sub>4</sub> is a linear combination of other vectors in the set.

- B.  $\{u_1, u_2, u_3, u_4\}$  could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- C.  $\{u_1, u_2, u_3, u_4\}$  is always a linearly independent set of vectors.
- D. {u<sub>1</sub>, u<sub>2</sub>, u<sub>3</sub>, u<sub>4</sub>} is always a linearly dependent set of vectors.
- E.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  is a linearly independent set of vectors unless  $\mathbf{u}_4 = \mathbf{0}$ .
- F. none of the above

# ${\bf 24.} \hspace{1.5cm} (1\ pt) \ Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/2.3.47.pg$

Let  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  be a linearly independent set of vectors. Select the best statement.

- A.  $\{u_1, u_2, u_3\}$  could be a linearly independent or linearly dependent set of vectors depending on the vectors chosen.
- B. {u<sub>1</sub>, u<sub>2</sub>, u<sub>3</sub>} is never a linearly independent set of vectors.
- C. {u<sub>1</sub>, u<sub>2</sub>, u<sub>3</sub>} is always a linearly independent set of vectors.
- D. none of the above

### ${\bf 25.} \qquad (1\ \ pt)\ \ Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/3.3.42.pg$

A must be a square matrix to be invertible. ?

# ${\bf 26.} \qquad (1\ \ pt)\ \ Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/4.1.22.pg$

Find the null space for  $A = \begin{bmatrix} 1 & 2 \\ 3 & 8 \end{bmatrix}$ . What is null(A)?

- A. span  $\left\{ \begin{bmatrix} 1\\3 \end{bmatrix} \right\}$ • B. span  $\left\{ \begin{bmatrix} -2\\1 \end{bmatrix} \right\}$
- C. ℝ<sup>2</sup>
- D.  $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- E. span  $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$
- F. span  $\left\{ \begin{bmatrix} -3\\1 \end{bmatrix} \right\}$
- G. span  $\left\{ \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$
- H. none of the above

# ${\bf 27.} \qquad (1\ \ pt)\ \ Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/4.1.27.pg$

Find the null space for  $A = \begin{bmatrix} 3 & -4 \\ 2 & 2 \\ 1 & -7 \end{bmatrix}$ .

What is null(A)?

- A.  $\mathbb{R}^2$ • B. span  $\left\{ \begin{bmatrix} 3 \\ -4 \end{bmatrix} \right\}$
- C. span  $\left\{ \begin{bmatrix} +4\\3 \end{bmatrix} \right\}$
- D.  $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- E. span  $\left\{ \begin{bmatrix} 3\\2\\1 \end{bmatrix} \right\}$
- F.  $\mathbb{R}^3$
- G. span  $\left\{ \begin{bmatrix} -4\\3 \end{bmatrix} \right\}$
- H. span  $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$
- I. none of the above

## ${\bf 28.} \qquad (1\ pt)\ Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/4.1.28.pg$

Find the null space for  $A = \begin{bmatrix} 1 & -2 \\ 4 & -8 \\ -7 & 14 \end{bmatrix}$ .

What is null(A)?

- A. span  $\left\{ \begin{bmatrix} 1\\4\\-7 \end{bmatrix} \right\}$
- B.  $\mathbb{R}^2$
- C. span  $\left\{ \begin{bmatrix} +2\\1 \end{bmatrix} \right\}$
- D. span  $\left\{ \begin{bmatrix} 0\\0\\0 \end{bmatrix} \right\}$
- E. span  $\left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$
- F. span  $\left\{ \begin{bmatrix} 14\\-7 \end{bmatrix} \right\}$
- G. ℝ
- H. none of the above

# 29. (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/4.1.30.pg

Find the null space for 
$$A = \begin{bmatrix} 3 & -1 & 3 \\ -6 & -3 & -21 \\ 2 & 2 & 10 \end{bmatrix}$$
.

What is null(A)?

• A. span 
$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -6 \\ -3 \\ -21 \end{bmatrix} \right\}$$

• B. 
$$\mathbb{R}^3$$

• C. span 
$$\left\{ \begin{bmatrix} 3 \\ -1 \\ 3 \end{bmatrix} \right\}$$

• D. span 
$$\left\{ \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix} \right\}$$

• E. span 
$$\left\{ \begin{bmatrix} 3 \\ -6 \\ 2 \end{bmatrix} \right\}$$

• F. span 
$$\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

• G. none of the above

# 30. (1 pt) Library/WHFreeman/Holt\_linear\_algebra/Chaps\_1-4-/4.3.47.pg

Indicate whether the following statement is true or false.

? 1. If A and B are equivalent matrices, then col(A) = col(B).

#### 31. (1 pt) Library/maCalcDB/setLinearAlgebra3Matrices/ur\_la\_3\_6.pg

If A and B are  $3 \times 6$  matrices, and C is a  $7 \times 3$  matrix, which of the following are defined?

- A. BC
- B.  $BA^T$
- C. C<sup>T</sup>
- D. *CA*
- E. B+A
- F. A + C

# ${\bf 32.} \qquad (1\ pt)\ Library/maCalcDB/setLinearAlgebra4InverseMatrix-/ur\_la\_4\_8.pg$

Determine which of the formulas hold for all invertible  $n \times n$  matrices A and B

- A. A + B is invertible
- B.  $(AB)^{-1} = A^{-1}B^{-1}$
- C. AB = BA

- D.  $(A+B)^2 = A^2 + B^2 + 2AB$
- E.  $A^6$  is invertible
- F.  $(I_n + A)(I_n + A^{-1}) = 2I_n + A + A^{-1}$

#### 33. (1 pt) UI/Fall14/lin\_span2.pg

Which of the following sets of vectors span  $\mathbb{R}^3$ ?

• A. 
$$\begin{bmatrix} -3 \\ -8 \\ -4 \end{bmatrix}$$
,  $\begin{bmatrix} 0 \\ 5 \\ -6 \end{bmatrix}$ ,  $\begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix}$ 

• C. 
$$\begin{bmatrix} -5 \\ 4 \\ 8 \end{bmatrix}$$
,  $\begin{bmatrix} 7 \\ 5 \\ -9 \end{bmatrix}$ ,  $\begin{bmatrix} -12 \\ -1 \\ 17 \end{bmatrix}$ 

• D. 
$$\begin{vmatrix} 8 \\ -4 \end{vmatrix}$$
,  $\begin{vmatrix} 4 \\ -2 \end{vmatrix}$ 

• E. 
$$\begin{bmatrix} 8 \\ 7 \end{bmatrix}$$
,  $\begin{bmatrix} 5 \\ -9 \end{bmatrix}$ ,  $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ 

• F. 
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
,  $\begin{bmatrix} -5 \\ 9 \end{bmatrix}$ 

Which of the following sets of vectors are linearly independent?

• A. 
$$\begin{bmatrix} -3 \\ -8 \\ -4 \end{bmatrix}$$
,  $\begin{bmatrix} 0 \\ 5 \\ -6 \end{bmatrix}$ ,  $\begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix}$ 

• C. 
$$\begin{vmatrix} -5 \\ 4 \\ 8 \end{vmatrix}$$
,  $\begin{vmatrix} 7 \\ 5 \\ -9 \end{vmatrix}$ ,  $\begin{vmatrix} -12 \\ -1 \\ 17 \end{vmatrix}$ 

• D. 
$$\begin{vmatrix} 8 \\ -4 \end{vmatrix}$$
,  $\begin{vmatrix} 4 \\ -2 \end{vmatrix}$ 

• E. 
$$\begin{bmatrix} 8 \\ 7 \end{bmatrix}$$
,  $\begin{bmatrix} 5 \\ -9 \end{bmatrix}$ ,  $\begin{bmatrix} -4 \\ 2 \end{bmatrix}$ 

• F. 
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
,  $\begin{bmatrix} -5 \\ 9 \end{bmatrix}$ 

34. (1 pt) UI/Fall14/lin\_span.pg

Let 
$$A = \begin{bmatrix} 14 \\ 3 \\ 32 \end{bmatrix}$$
,  $B = \begin{bmatrix} -2 \\ 1 \\ -6 \end{bmatrix}$ , and  $C = \begin{bmatrix} -2 \\ -1 \\ -4 \end{bmatrix}$ 

Which of the following best describes the span of the above 3 vectors?

- A. 0-dimensional point in  $\mathbb{R}^3$
- B. 1-dimensional line in  $\mathbb{R}^3$
- C. 2-dimensional plane in  $\mathbb{R}^3$
- D. R<sup>3</sup>

Determine whether or not the three vectors listed above are linearly independent or linearly dependent.

- A. linearly dependent
- B. linearly independent

If they are linearly dependent, determine a non-trivial linear relation. Otherwise, if the vectors are linearly independent, enter 0's for the coefficients, since that relationship **always** holds.

$$A + B + C = 0.$$

#### 35. (1 pt) local/Library/Rochester/setLinearAlgebra3Matrices-/ur\_la\_3\_14.pg

Find the ranks of the following matrices.

$$rank \left[ \begin{array}{cccc} 0 & 6 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 \end{array} \right] = \underline{\hspace{1cm}}$$

36. (1 pt) local/Library/TCNJ/TCNJ\_BasesLinearlyIndependentSet-/3.pg

Check the true statements below:

- A. If B is an echelon form of a matrix A, then the pivot columns of B form a basis for ColA.
- B. A basis is a spanning set that is as large as possible.
- C. The column space of a matrix A is the set of solutions of Ax = b.
- D. The columns of an invertible  $n \times n$  matrix form a basis for  $\mathbb{R}^n$ .
- E. If  $H = Span\{b_1, ..., b_p\}$ , then  $\{b_1, ..., b_p\}$  is a basis

#### 37. (1 pt) local/Library/TCNJ/TCNJ\_LinearSystems/problem6.pg Give a geometric description of the following systems of equations

Hint: (Instructor hint preview: show the student hint after 1 attempts. The current number of attempts is 0.)

Reduce the augmented matrix and solve for it. If it has unique solutions, three planes intersect at a point; no solutions indicates no common intersection; one free variable shows intersection on a line; two free variables means identical planes.

#### 38. (1 pt) local/Library/UI/2.3.49.pg

Let  $\mathbf{u}_4$  be a linear combination of  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ . Select the best statement.

- A.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  is always a linearly independent set of vectors.
- B.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  is never a linearly dependent set of vec-
- C.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  is never a linearly independent set of
- D.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4\}$  could be a linearly dependent or linearly independent set of vectors depending on the vectors chosen.
- E.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  is a linearly dependent set of vectors.
- F.  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  is a linearly dependent set of vectors unless one of  $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$  is the zero vector.
- G. none of the above

#### **39.** (1 pt) local/Library/UI/4.1.23.pg

Find the null space for  $A = \begin{bmatrix} 1 & 0 & -6 \\ 0 & 1 & 3 \end{bmatrix}$ . What is null(A)?

• A. 
$$\mathbb{R}^3$$
  
• B. span  $\left\{ \begin{bmatrix} 1\\0\\+6 \end{bmatrix}, \begin{bmatrix} 0\\1\\-3 \end{bmatrix} \right\}$   
• C. span  $\left\{ \begin{bmatrix} -3\\+6 \end{bmatrix} \right\}$ 

• D. 
$$\mathbb{R}^2$$
  
• E. span  $\left\{ \begin{bmatrix} +6 \\ -3 \\ 1 \end{bmatrix} \right\}$   
• F. span  $\left\{ \begin{bmatrix} +6 \\ -3 \end{bmatrix} \right\}$   
• G. span  $\left\{ \begin{bmatrix} -3 \\ +6 \end{bmatrix} \right\}$ 

H. none of the above

#### 40. (1 pt) local/Library/UI/Fall14/HW7\_4.pg

Determine if the subset of  $\mathbb{R}^2$  consisting of vectors of the form  $\begin{bmatrix} a \\ b \end{bmatrix}$ , where a and b are integers, is a subspace.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

#### 41. (1 pt) local/Library/UI/Fall14/HW7\_5.pg

Determine if the subset of  $\mathbb{R}^3$  consisting of vectors of the

form 
$$\begin{bmatrix} a \\ b \\ c \end{bmatrix}$$
, where  $a \ge 0$ ,  $b \ge 0$ , and  $c \ge 0$  is a subspace.

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

#### 42. (1 pt) local/Library/UI/Fall14/HW7\_6.pg

If *A* is an  $n \times n$  matrix and  $\mathbf{b} \neq 0$  in  $\mathbb{R}^n$ , then consider the set of solutions to  $A\mathbf{x} = \mathbf{b}$ .

Select true or false for each statement.

The set contains the zero vector

- A. True
- B. False

This set is closed under vector addition

- A. True
- B. False

This set is closed under scalar multiplications

- A. True
- B. False

This set is a subspace

- A. True
- B. False

#### 43. (1 pt) local/Library/UI/Fall14/HW7\_11.pg

Find all values of x for which rank(A) = 2.

$$A = \begin{bmatrix} 2 & 2 & 0 & 5 \\ 4 & 7 & x & 19 \\ -6 & -9 & -3 & -24 \end{bmatrix}$$

- A. -4
  - B. -3
  - C. -2
  - D. -1
  - E. 0
  - F. 1
  - G. 2
  - H. 3
  - I. 4
  - J. none of the above

#### 44. (1 pt) local/Library/UI/Fall14/HW7\_12.pg

Suppose that A is a  $5 \times 6$  matrix which has a null space of dimension 6. The rank of A=

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

#### 45. (1 pt) local/Library/UI/Fall14/HW7\_25.pg

Indicate whether the following statement is true or false? If  $S = \text{span}u_1, u_2, u_3$ , then dim(S) = 3.

- A. True
- B. False

#### 46. (1 pt) local/Library/UI/Fall14/HW7\_27.pg

Determine the rank and nullity of the matrix.

The rank of the matrix is

- A. -4
- B. -3
- C. -2
- D. -1

- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

The nullity of the matrix is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

#### 47. (1 pt) local/Library/UI/Fall14/HW8\_2.pg

Evaluate the following  $3 \times 3$  determinant. Use the properties of determinants to your advantage.

$$\begin{vmatrix} -8 & 0 & -4 \\ -1 & 0 & 10 \\ -4 & 0 & 3 \end{vmatrix}$$

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Does the matrix have an inverse?

- A. No
- B. Yes

#### 48. (1 pt) local/Library/UI/Fall14/HW8\_3.pg

0 -2 3Given the matrix

- (a) find its determinant
  - A. 6
  - B. -5
  - C. -4

- D. -2
- E. -1
- F. 0
- G. 1
- H. 3
- I. 5
- J. 7
- K. None of those above
- (b) Does the matrix have an inverse?
  - A. No
  - B. Yes

#### 49. (1 pt) local/Library/UI/Fall14/HW8\_4.pg

If A and B are  $4 \times 4$  matrices, det(A) = 1, det(B) = 2, then det(AB) =

- A. -15
- B. 2
- C. -11
- D. -8
- E. -5
- F. 0 • G. 3
- H. 6
- I. 8
- J. 12
- K. None of those above

$$\det(-3A) =$$

- A. -40
- B. 81
- C. -28
- D. -21
- E. -10 • F. -1
- G. 10
- H. 21 • I. 28
- J. 36
- K. 40
- L. None of those above

$$\det(A^T) =$$

- A. -3
- B. -2
- C. -1
- D. 0

- E. 1
- F. 2
- G. 3
- H. 4
- I. None of those above

 $\det(B^{-1}) =$ 

- A. -0.4
- B. -0.5
- C. 0
- D. 0.4
- E. 0.5
- F. 1
- G. None of those above

 $det(B^4) =$ 

- A. -81
- B. -36
- C. -12
- D. 0
- E. 12
- F. 36
- G. 81
- H. 16
- I. 1024
- J. None of those above

50. (1 pt) local/Library/UI/Fall14/HW8\_5.pg

Find the determinant of the matrix

$$A = \begin{bmatrix} -6 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ -8 & 1 & 3 & 0 \\ -6 & -3 & 1 & 3 \end{bmatrix}$$

$$\det(A) = \begin{bmatrix} -6 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 \\ -6 & -3 & 1 & 3 \end{bmatrix}$$

- - A. -400B. -360
  - C. -288
  - D. -120
  - E. 0
  - F. 120
  - G. -108
  - H. 240
  - I. 360
  - J. 400
  - K. None of those above

51. (1 pt) local/Library/UI/Fall14/HW8\_7.pg

Suppose that a  $4 \times 4$  matrix A with rows  $v_1$ ,  $v_2$ ,  $v_3$ , and  $v_4$  has determinant  $\det A = -4$ . Find the following determinants:

$$B = \begin{bmatrix} v_1 \\ 9v_2 \\ v_3 \\ v_4 \end{bmatrix} \det(B) =$$

- A. -18
- B. -15
- C. -12
- D. -36
- E. -9
- F. 0
- G. 9
- H. 12I. 15
- J. 18
- K. None of those above

$$C = \begin{bmatrix} v_2 \\ v_1 \\ v_4 \\ v_3 \end{bmatrix} \det(C) =$$

- A. -18
- B. -4
- C. -9
- D. -3
- E. 0
- F. 3
- G. 9 • H. 12
- I. 18
- J. None of those above

$$D = \begin{bmatrix} v_1 + 3v_3 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$$
$$\det(D) =$$

- A. -18
- B. -4
- C. -9
- D. -3
- E. 0
- F. 3
- G. 9
- H. 12
- I. 18
- J. None of those above

#### 52. (1 pt) local/Library/UI/Fall14/HW8\_8.pg

Use determinants to determine whether each of the following sets of vectors is linearly dependent or independent.

$$\left[\begin{array}{c} -9 \\ 6 \end{array}\right], \left[\begin{array}{c} 4 \\ -5 \end{array}\right],$$

- A. Linearly Dependent
- B. Linearly Independent

$$\begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ -18 \\ -1 \end{bmatrix}, \begin{bmatrix} 7 \\ 44 \\ -1 \end{bmatrix},$$

- A. Linearly Dependent
- B. Linearly Independent

$$\left[\begin{array}{c} -5 \\ -10 \end{array}\right], \left[\begin{array}{c} 3 \\ 6 \end{array}\right],$$

- A. Linearly Dependent
- B. Linearly Independent

$$\left[\begin{array}{c}2\\4\\4\end{array}\right],\left[\begin{array}{c}-4\\-8\\-8\end{array}\right],\left[\begin{array}{c}10\\20\\20\end{array}\right],$$

- A. Linearly Dependent
- B. Linearly Independent

#### 53. (1 pt) local/Library/UI/Fall14/HW8\_10.pg

$$A = \left[ \begin{array}{rrrr} -8 & 6 & 0 & 3 \\ -4 & -2 & 0 & 0 \\ 0 & -7 & 0 & 0 \\ 8 & -2 & -4 & 8 \end{array} \right]$$

The determinant of the matrix is

- A. -1890
- B. -1024
- C. -630
- D. 336
- E. -210
- F. 0
- G. 324
- H. 630
- I. 1024
- J. None of those above

Hint: Find a good row or column and expand by minors.

#### 54. (1 pt) local/Library/UI/Fall14/HW8\_11.pg

Find the determinant of the matrix

$$M = \begin{bmatrix} -3 & 0 & 0 & -2 & 0 \\ -3 & 0 & 3 & 0 & 0 \\ 0 & -3 & 0 & 0 & -2 \\ 0 & 0 & 0 & -3 & 2 \\ 0 & -2 & -2 & 0 & 0 \end{bmatrix}.$$

$$\det(M) =$$

- A. -48
- B. -35
- C. -20
- D. 180
- E. -5
- F. 5
- G. 18
- H. 20
- I. 81
- J. None of those above

#### 55. (1 pt) local/Library/UI/Fall14/HW8\_12.pg

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

And now for the grand finale: The determinant of the matrix is

- A. -362880
- B. -5
- C. 0
- D. 5
- E. 20
- F. 30
- G. 40
- H. 362880
- I. None of the above

**Hint:** Remember that a square linear system has a unique solution if the determinant of the coefficient matrix is non-zero.

A system of equations can have exactly 2 solution.

- A. True
- B. False

A system of linear equations can have exactly 2 solution.

- A. True
- B. False

A system of linear equations has no solution if and only if the last column of its augmented matrix corresponds to a pivot column.

- A. True
- B. False

A system of linear equations has an infinite number of solutions if and only if its associated augmented matrix has a column corresponding to a free variable.

- A. True
- B. False

If a system of linear equations has an infinite number of solutions, then its associated augmented matrix has a column corresponding to a free variable.

- A. True
- B. False

If a linear system has four equations and seven variables, then it must have infinitely many solutions.

- A. True
- B. False

Every linear system with free variables has infinitely many solutions.

- A. True
- B. False

Any linear system with more variables than equations cannot have a unique solution.

- A. True
- B. False

If a linear system has the same number of equations and variables, then it must have a unique solution.

- A. True
- B. False

A vector b is a linear combination of the columns of a matrix A if and only if the equation Ax = b has at least one solution.

- A. True
- B. False

If the columns of an  $m \times n$  matrix, A span  $\mathbb{R}^m$ , then the equation, Ax = b is consistent for each b in  $\mathbb{R}^m$ .

- A. True
- B. False

If A is an  $m \times n$  matrix and if the equation Ax = b is inconsistent for some b in  $\mathbb{R}^m$ , then A cannot have a pivot position in every row.

- A. True
- B. False

Any linear combination of vectors can always be written in the form Ax for a suitable matrix A and vector x.

- A. True
- B. False

If the equation Ax = b is inconsistent, then b is not in the set spanned by the columns of A.

- A. True
- B. False

If A is an  $m \times n$  matrix whose columns do not span  $\mathbb{R}^m$ , then the equation Ax = b is inconsistent for some b in  $\mathbb{R}^m$ .

- A. True
- B. False

Every linear system with free variables has infinitely many solutions.

- A. True
- B. False

#### 72. (1 pt) local/Library/UI/Fall14/quiz2\_2.pg

Find the area of the triangle with vertices (1, -2), (8, -5), and (3, 2).

Area =

- A. 2
- B. 5
- C. 6
- D. 8
- E. 9
- F. 12
- G. 17
- H. 20
- I. 25
- J. None of those above

Hint: The area of a triangle is half the area of a parallelogram. Find the vectors that determine the parallelogram of interest. If you have difficulty, visualizing the problem may be helpful: plot the 3 points.

#### 73. (1 pt) local/Library/UI/Fall14/quiz2\_6.pg

Determine if v is an eigenvector of the matrix A.

$$1. A = \begin{bmatrix} 8 & -4 & 15 \\ -7 & 5 & -15 \\ -6 & 4 & -13 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$$

- A. Yes
- B. No

$$2. A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -2 & 0 \\ 5 & -1 & -1 \end{bmatrix}, v = \begin{bmatrix} 6 \\ 6 \\ -1 \end{bmatrix}$$

- A. Yes
- B. No

$$3. A = \begin{bmatrix} -8 & -4 & -1 \\ 13 & 3 & -5 \\ -10 & -4 & 1 \end{bmatrix}, v = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

- A. Yes
- B. No

$$4. A = \begin{bmatrix} 6 & 5 & -16 \\ 6 & 5 & -16 \\ 3 & 5 & -13 \end{bmatrix}, v = \begin{bmatrix} 7 \\ 9 \\ 1 \end{bmatrix}$$

- A. Yes
- B. No

#### 74. (1 pt) local/Library/UI/Fall14/quiz2\_7.pg

Given that  $v_1 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$  and  $v_2 = \begin{bmatrix} 8 \\ 3 \end{bmatrix}$  are eigenvectors of the matrix  $A = \begin{bmatrix} 62 & -168 \\ 21 & -57 \end{bmatrix}$ , determine the corresponding eigenvalues. a.  $\lambda_1 =$ 

- A. -6
- B. -5
- C. -4
- D. -3
- E. -2
- F. -1
- G. 6
- H. 0
- I. 1
- J. 2
- K. None of those above

b. 
$$\lambda_2 =$$

- A. -5
- B. -4
- C. -3
- D. -2
- E. 0
- F. -1
- G. 1
- H. 2
- I. 3
- J. None of those above

#### 75. (1 pt) local/Library/UI/Fall14/volume1.pg

Find the volume of the parallelepiped determined by vectors

$$\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -4 \\ 0 \end{bmatrix}, \text{ and } \begin{bmatrix} -2 \\ -2 \\ -5 \end{bmatrix}$$

- A. 72
- B. -5
- C. -4
- D. -2
- E. -1
- F. 0
- G. 1
- H. 3
- I. 5
- J. 7
- K. None of those above

Suppose a 3 x 5 augmented matrix contains a pivot in every row. Then the corresponding system of equations has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution, or an infinite number of solutions, depending on the system of equations
- H. none of the above

Given the following augmented matrix,

$$A = \left[ \begin{array}{ccccc} 2 & 0 & 3 & -2 & 4 \\ 0 & -5 & 4 & -6 & 4 \\ 0 & 0 & 0 & 0 & 5 \end{array} \right],$$

the corresponding system of equations has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Given the following augmented matrix,

$$A = \left[ \begin{array}{rrrrr} -1 & -3 & 3 & -3 & 7 \\ 0 & -3 & 3 & 3 & 2 \\ 0 & 0 & 0 & -2 & -8 \end{array} \right],$$

the corresponding system of equations has

• A. No solution

- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Suppose an augmented matrix contains a pivot in the last column. Then the corresponding system of equations has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Suppose a coefficient matrix A contains a pivot in the last column. Then  $A\vec{x} = \vec{b}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Suppose a coefficient matrix A contains a pivot in every row. Then  $A\vec{x} = \vec{b}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Suppose a coefficient matrix *A* contains a pivot in every column. Then  $A\vec{x} = \vec{b}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Suppose  $A\vec{x} = \vec{0}$  has an infinite number of solutions, then given a vector  $\vec{b}$  of the appropriate dimension,  $A\vec{x} = \vec{b}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Suppose  $A\vec{x} = \vec{b}$  has an infinite number of solutions, then  $A\vec{x} = \vec{0}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Suppose  $A\vec{x} = \vec{0}$  has a unique solution, then given a vector  $\vec{b}$  of the appropriate dimension,  $A\vec{x} = \vec{b}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations

#### • H. none of the above

Suppose  $A\vec{x} = \vec{b}$  has a unique solution, then  $A\vec{x} = \vec{0}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Suppose  $A\vec{x} = \vec{b}$  has no solution, then  $A\vec{x} = \vec{0}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Suppose A is a square matrix and  $A\vec{x}=\vec{0}$  has a unique solution, then given a vector  $\vec{b}$  of the appropriate dimension,  $A\vec{x}=\vec{b}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions
- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

Suppose A is a square matrix and  $A\vec{x} = \vec{0}$  has an infinite number of solutions, then given a vector  $\vec{b}$  of the appropriate dimension,  $A\vec{x} = \vec{b}$  has

- A. No solution
- B. Unique solution
- C. Infinitely many solutions
- D. at most one solution
- E. either no solution or an infinite number of solutions
- F. either a unique solution or an infinite number of solutions

- G. no solution, a unique solution or an infinite number of solutions, depending on the system of equations
- H. none of the above

#### 90. (1 pt) local/Library/UI/LinearSystems/spanHW4.pg

Let 
$$A = \begin{bmatrix} -5 & 5 & 0 \\ 5 & -7 & 4 \\ -3 & 5 & -3 \end{bmatrix}$$
, and  $b = \begin{bmatrix} -2 \\ 2 \\ -3 \end{bmatrix}$ .

Denote the columns of A by  $a_1$ ,  $a_2$ ,  $a_3$ , and let  $W = span\{a_1, a_2, a_3\}$ .

- ? 1. Determine if b is in  $\{a_1, a_2, a_3\}$
- ? 2. Determine if *b* is in *W*

How many vectors are in  $\{a_1, a_2, a_3\}$ ? (For infinitely many, enter -1) \_\_\_\_\_

How many vectors are in W? (For infinitely many, enter -1)

#### $\bf 91.\ (1\ pt)\ local/Library/UI/MatrixAlgebra/Euclidean/2.3.43.pg$

Let *A* be a matrix with linearly independent columns. Select the best statement.

- A. The equation  $A\mathbf{x} = \mathbf{0}$  has nontrivial solutions precisely when it has more rows than columns.
- B. The equation  $A\mathbf{x} = \mathbf{0}$  has nontrivial solutions precisely when it has more columns than rows.
- C. The equation  $A\mathbf{x} = \mathbf{0}$  has nontrivial solutions precisely when it is a square matrix.
- D. The equation  $A\mathbf{x} = \mathbf{0}$  always has nontrivial solutions.
- E. There is insufficient information to determine if such an equation has nontrivial solutions.
- F. The equation  $A\mathbf{x} = \mathbf{0}$  never has nontrivial solutions.
- G. none of the above

#### 92. (1 pt) local/Library/UI/MatrixAlgebra/Euclidean/2.3.44.pg

Let *A* be a matrix with linearly independent columns. Select the best statement.

- A. The equation  $A\mathbf{x} = \mathbf{b}$  has a solution for all  $\mathbf{b}$  precisely when it has more rows than columns.
- B. The equation  $A\mathbf{x} = \mathbf{b}$  has a solution for all  $\mathbf{b}$  precisely when it has more columns than rows.
- C. There is insufficient information to determine if  $A\mathbf{x} = \mathbf{b}$  has a solution for all  $\mathbf{b}$ .
- D. The equation  $A\mathbf{x} = \mathbf{b}$  has a solution for all  $\mathbf{b}$  precisely when it is a square matrix.
- E. The equation  $A\mathbf{x} = \mathbf{b}$  never has a solution for all  $\mathbf{b}$ .
- F. The equation  $A\mathbf{x} = \mathbf{b}$  always has a solution for all  $\mathbf{b}$ .
- G. none of the above

#### 93. (1 pt) local/Library/UI/problem7.pg

A and B are  $n \times n$  matrices.

Adding a multiple of one row to another does not affect the determinant of a matrix.

- A. True
- B. False

If the columns of A are linearly dependent, then det A = 0.

- A. True
- B. False

$$det(A+B) = detA + detB$$
.

- A. True
- B. False

Suppose A is a 8  $\times$  6 matrix. If rank of A = 2, then nullity of A = 2

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3I. 4
- J. none of the above

The vector  $\vec{b}$  is NOT in ColA if and only if  $A\vec{v} = \vec{b}$  does NOT have a solution

- A. True
- B. False

The vector  $\vec{b}$  is in *ColA* if and only if  $A\vec{v} = \vec{b}$  has a solution

- A. True
- B. False

The vector  $\vec{v}$  is in *NulA* if and only if  $A\vec{v} = \vec{0}$ 

• A. True

• B. False

If the equation  $A\vec{x} = \vec{b_1}$  has at least one solution and if the equation  $A\vec{x} = \vec{b_2}$  has at least one solution, then the equation  $A\vec{x} = 8\vec{b_1} - 7\vec{b_2}$  also has at least one solution.

• A. True

• B. False

**Hint:** (Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.) Is *colA* a subspace? Is *colA* closed under linear combinations?

If  $\vec{x_1}$  and  $\vec{x_2}$  are solutions to  $A\vec{x} = \vec{0}$ , then  $-5\vec{x_1} - 8\vec{x_2}$  is also a solution to  $A\vec{x} = \vec{0}$ .

• A. True

• B. False

**Hint:** (Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)

Is NulA a subspace? Is NulA closed under linear combinations?

If  $\vec{x_1}$  and  $\vec{x_2}$  are solutions to  $A\vec{x} = \vec{b}$ , then  $-1\vec{x_1} + 1\vec{x_2}$  is also a solution to  $A\vec{x} = \vec{b}$ .

• A. True

• B. False

Hint: (Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0. )

Is the solution set to  $A\vec{x} = \vec{b}$  a subspace even when  $\vec{b}$  is not  $\vec{0}$ ? Is the solution set to  $A\vec{x} = \vec{b}$  closed under linear combinations even when  $\vec{b}$  is not  $\vec{0}$ ?

Find the area of the parallelogram determined by the vectors

A. -4

• B. -3

• C. -2

• D. -1

• E. 0

• F. 1 • G. 2

• H. 3 • I. 4

• J. 5

Suppose A is a 8  $\times$  4 matrix. Then nul A is a subspace of  $R^k$ where k =

• A. -4

• B. -3

• C. -2

• D. -1

• E. 0

• F. 1

• G. 2

• H. 3 • I. 4

• J. none of the above

Suppose A is a 3  $\times$  7 matrix. Then col A is a subspace of  $R^k$ where k =

• A. -4

• B. -3

• C. -2

• D. -1

• E. 0

• F. 1 • G. 2

 H. 3 • I. 4

• J. none of the above

Calculate the determinant of

• A. -4

• B. -3

• C. -2

• D. -1

• E. 0

• F. 1

• G. 2 • H. 3

• I. 4

• J. 5

Suppose 
$$A \begin{bmatrix} -5 \\ -3 \\ -4 \end{bmatrix} = \begin{bmatrix} 20 \\ 12 \\ 16 \end{bmatrix}$$
. Then an eigenvalue of  $A$  is

A. -4

• B. -3

C. -2

D. -1 • E. 0

• F. 1

• G. 2

- H. 3
- I. 4
- J. none of the above

#### 106. (1 pt) local/Library/UI/volumn2.pg

A and B are  $n \times n$  matrices.

Check the true statements below:

- A. If A is 3x3, with columns  $a_1$ ,  $a_2$ ,  $a_3$ , then detA equals the volume of the parallelpiped determined by the vectors  $a_1$ ,  $a_2$ ,  $a_3$ .
- B. If A is 3x3, with columns  $a_1$ ,  $a_2$ ,  $a_3$ , then the absolute value of detA equals the volume of the parallelpiped determined by the vectors  $a_1$ ,  $a_2$ ,  $a_3$ .
- C.  $det A^T = (-1) det A$ .

### 107. (1 pt) UI/DIAGtfproblem1.pg

A, P and D are  $n \times n$  matrices.

Check the true statements below:

- A. A is diagonalizable if A has n distinct linearly independent eigenvectors.
- B. If AP = PD, with D diagonal, then the nonzero columns of P must be eigenvectors of A.
- C. A is diagonalizable if  $A = PDP^{-1}$  for some diagonal matrix D and some invertible matrix P.
- D. A is diagonalizable if and only if A has n eigenvalues, counting multiplicities.
- E. If there exists a basis for  $\mathbb{R}^n$  consisting entirely of eigenvectors of A, then A is diagonalizable.
- F. A is diagonalizable if A has n distinct eigenvectors.
- G. If *A* is diagonalizable, then *A* has *n* distinct eigenvalues.
- H. If *A* is symmetric, then *A* is orthogonally diagonalizable.
- I. If A is orthogonally diagonalizable, then A is symmetric.
- J. If A is diagonalizable, then A is invertible.
- K. If A is symmetric, then A is diagonalizable.
- L. If *A* is invertible, then *A* is diagonalizable.
- M. If A is diagonalizable, then A is symmetric.

#### **108.** (1 pt) UI/orthog.pg

All vectors and subspaces are in  $\mathbb{R}^n$ .

Check the true statements below:

- A. If  $A\mathbf{v} = r\mathbf{v}$  and  $A\mathbf{w} = s\mathbf{w}$  and  $r \neq s$ , then  $\mathbf{v} \cdot \mathbf{w} = 0$ .
- B. If  $W = Span\{x_1, x_2, x_3\}$  and if  $\{v_1, v_2, v_3\}$  is an orthonormal set in W, then  $\{v_1, v_2, v_3\}$  is an orthonormal basis for W.

- C. In a QR factorization, say A = QR (when A has linearly independent columns), the columns of Q form an orthonormal basis for the column space of A.
- D. If x is not in a subspace W, then  $x \text{proj}_W(x)$  is not zero.
- E. If  $\{v_1, v_2, v_3\}$  is an orthonormal set, then the set  $\{v_1, v_2, v_3\}$  is linearly independent.
- F. If  $\mathbf{v}$  and  $\mathbf{w}$  are both eigenvectors of A and if A is symmetric, then  $\mathbf{v} \cdot \mathbf{w} = 0$ .
- G. If A is symmetric,  $A\mathbf{v} = r\mathbf{v}$ ,  $A\mathbf{w} = s\mathbf{w}$  and  $r \neq s$ , then  $\mathbf{v} \cdot \mathbf{w} = 0$ .

#### 109. (1 pt) local/Library/UI/eigenTF.pg

A is  $n \times n$  an matrices.

Check the true statements below:

- A. The vector **0** can never be an eigenvector of A
- B. 0 can never be an eigenvalue of A.
- C. The eigenspace corresponding to a particular eigenvalue of *A* contains an infinite number of vectors.
- D. 0 is an eigenvalue of *A* if and only if the columns of *A* are linearly dependent.
- E. A will have at most n eigenvectors.
- F. The vector  $\mathbf{0}$  is an eigenvector of A if and only if Ax = 0 has a nonzero solution
- G. There are an infinite number of eigenvectors that correspond to a particular eigenvalue of A.
- H. 0 is an eigenvalue of A if and only if det(A) = 0
- I. A will have at most n eigenvalues.
- J. 0 is an eigenvalue of A if and only if Ax = 0 has a nonzero solution
- K. The vector **0** is an eigenvector of *A* if and only if the columns of *A* are linearly dependent.
- L. The vector **0** is an eigenvector of *A* if and only if det(A) = 0
- M. 0 is an eigenvalue of A if and only if Ax = 0 has an infinite number of solutions

Suppose  $A = PDP^{-1}$  where D is a diagonal matrix. If  $P = [\vec{p_1} \ \vec{p_2} \ \vec{p_3}]$ , then  $2\vec{p_1}$  is an eigenvector of A

- A. True
- B. False

Suppose  $A = PDP^{-1}$  where D is a diagonal matrix. If  $P = [\vec{p_1} \ \vec{p_2} \ \vec{p_3}]$ , then  $\vec{p_1} + \vec{p_2}$  is an eigenvector of A

- A. True
- B. False

Suppose  $A=PDP^{-1}$  where D is a diagonal matrix. Suppose also the  $d_{ii}$  are the diagonal entries of D. If  $P=[\vec{p_1}\ \vec{p_2}\ \vec{p_3}]$  and  $d_{11}=d_{22}$ , then  $\vec{p_1}+\vec{p_2}$  is an eigenvector of A

- A. True
- B. False

Suppose  $A=PDP^{-1}$  where D is a diagonal matrix. Suppose also the  $d_{ii}$  are the diagonal entries of D. If  $P=[\vec{p_1} \vec{p_2} \vec{p_3}]$  and  $d_{22}=d_{33}$ , then  $\vec{p_1}+\vec{p_2}$  is an eigenvector of A

- A. True
- B. False

If  $\vec{v_1}$  and  $\vec{v_2}$  are eigenvectors of A corresponding to eigenvalue  $\lambda_0$ , then  $1\vec{v_1}+9\vec{v_2}$  is also an eigenvector of A corresponding to eigenvalue  $\lambda_0$  when  $1\vec{v_1}+9\vec{v_2}$  is not  $\vec{0}$ .

- A. True
- B. False

**Hint:** (*Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.* )

Is an eigenspace a subspace? Is an eigenspace closed under linear combinations?

Also, is  $1\vec{v_1} + 9\vec{v_2}$  nonzero?

Which of the following is an eigenvalue of  $\begin{bmatrix} -8 & -20 \\ 1 & 4 \end{bmatrix}$ .

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Let 
$$A = \begin{bmatrix} 9 & -1 & 1 \\ 0 & -8 & -4 \\ 0 & 0 & 9 \end{bmatrix}$$
. Is  $A =$  diagonalizable?

- A. yes
- B. no
- C. none of the above

Let 
$$A = \begin{bmatrix} -6 & -25 & 40 \\ 0 & -1 & -8 \\ 0 & 0 & -6 \end{bmatrix}$$
. Is  $A = \text{diagonalizable}$ ?

- A. yes
- B. no
- C. none of the above

Let 
$$A = \begin{bmatrix} 1 & 9 \\ 8 & -8 \end{bmatrix}$$
. Is  $A = \text{diagonalizable}$ ?

- A. yes
- B. no
- C. none of the above

**Hint:** (Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)

You do NOT need to do much work for this problem. You just need to know if the matrix *A* is diagonalizable. Since *A* is a 2 x 2 matrix, you need 2 linearly independent eigenvectors of *A* to form *P*. Does *A* have 2 linearly independent eigenvectors? Note you don't need to know what these eigenvectors are. You don't even need to know the eigenvalues.

Suppose  $A = PDP^{-1}$ . Then if  $d_{ii}$  are the diagonal entries of  $D, d_{11} =$ ,

- A. -4
- B. -3
- C. -2
- D. -1
- F 0
- F. 1
- G. 2
- H. 3I. 4
- J. 5

**Hint:** (Instructor hint preview: show the student hint after 0 attempts. The current number of attempts is 0.)

Use definition of eigenvalue since you know an eigenvector corresponding to eigenvalue  $d_{11}$ .

### $\begin{array}{r} -1 \\ -4 \\ \hline 2.66666666666667 \end{array}$ Calculate the dot product:

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

2 3.46410161513775 . Determine the length of

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. 5

If the characteristic polynomial of  $A = (\lambda - 6)^5(\lambda - 8)(\lambda +$ 9)8, then the algebraic multiplicity of  $\lambda = 8$  is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1 • G. 2
- H. 3
- I. 4
- J. none of the above

If the characteristic polynomial of  $A = (\lambda - 7)^7 (\lambda - 3)(\lambda +$ 8)<sup>3</sup>, then the geometric multiplicity of  $\lambda = 3$  is

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1

- G. 2
- H. 3
- I. 4
- J. none of the above

If the characteristic polynomial of  $A = (\lambda + 2)^5 (\lambda + 5)^2 (\lambda (6)^5$ , then the algebraic multiplicity of  $\lambda = -5$  is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2
- I. 0, 1, 2, or 3
- J. none of the above

If the characteristic polynomial of  $A = (\lambda + 1)^8 (\lambda + 4)^2 (\lambda +$  $(2)^8$ , then the geometric multiplicity of  $\lambda = -4$  is

- A. 0
- B. 1
- C. 2
- D. 3
- E. 0 or 1
- F. 0 or 2
- G. 1 or 2
- H. 0, 1, or 2 • I. 0, 1, 2, or 3
- J. none of the above

Suppose the orthogonal projection of is  $(z_1, z_2, z_3)$ . Then  $z_1 =$ 

- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

Suppose  $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$  is a unit vector in the direction of  $\begin{bmatrix} -4 \\ 4 \\ 3.52766841475279 \end{bmatrix}$ . Then  $u_1 =$ 

- A. -0.8
- B. -0.6
- C. -0.4
- D. -0.2
- E. 0
- F. 0.2
- G. 0.4
- H. 0.6
- I. 0.8
- J. 1

#### 128. (1 pt) Library/Rochester/setLinearAlgebra23QuadraticForms-/ur.la.23.2.pg

Find the eigenvalues of the matrix

$$M = \left[ \begin{array}{cc} 30 & 50 \\ 50 & 30 \end{array} \right].$$

Enter the two eigenvalues, separated by a comma:

Classify the quadratic form  $Q(x) = x^T Ax$ :

- A. Q(x) is positive semidefinite
- B. Q(x) is indefinite
- C. Q(x) is positive definite
- D. Q(x) is negative semidefinite
- E. Q(x) is negative definite

### 129. (1 pt) Library/Rochester/setLinearAlgebra23QuadraticForms/ur\_la\_23\_3.pg

The matrix

$$A = \begin{bmatrix} -6 & 0 & 0 \\ 0 & -1.4 & -0.8 \\ 0 & -0.8 & -2.6 \end{bmatrix}$$

has three distinct eigenvalues,  $\lambda_1 < \lambda_2 < \lambda_3$ ,

 $\lambda_1 = \underline{\hspace{1cm}},$ 

 $\lambda_2 = \underline{\hspace{1cm}}$ 

 $\lambda_3 = _{--}$ 

Classify the quadratic form  $Q(x) = x^T Ax$ :

- A. Q(x) is positive semidefinite
- B. Q(x) is negative semidefinite
- C. Q(x) is negative definite
- D. Q(x) is indefinite

#### • E. Q(x) is positive definite

Use Cramer's rule to solve the following system of equations for *x*:

$$-22x + 6y = 8$$

-4x + 1y = 0

- A. -4
- B. -3
- C. -2
- D. -1
- E. 0
- F. 1
- G. 2
- H. 3
- I. 4
- J. none of the above

#### 131. (1 pt) local/Library/UI/Fall14/quiz2\_9.pg

Supppose A is an invertible  $n \times n$  matrix and v is an eigenvector of A with associated eigenvalue 6. Convince yourself that v is an eigenvector of the following matrices, and find the associated eigenvalues:

- 1.  $A^5$ , eigenvalue =
  - A. 16
  - B. 81
  - C. 125
  - D. 216
  - E. 1024
  - F. 7776G. 2000
  - H. None of those above
- 2.  $A^{-1}$ , eigenvalue =
  - A. -0.5
  - B. -0.333
  - C. 0.166666666666667
  - D. -0.125
  - E. 0
  - F. 0.125
  - G. 0.333
  - H. 0.5
  - I. None of those above
- 3.  $A + 6I_n$ , eigenvalue =
  - A. -8
  - B. -4
  - C. -5
  - D. 0
  - E. 2

- F. 4
- G. 12
- H. 10
- I. None of those above

#### 4. 9A, eigenvalue =

- A. -40
- B. -36
- C. -28
- D. 54
- E.-12
- F. 0
- G. 24
- H. 36
- I. None of those above

### 132. (1 pt) local/Library/UI/Fall14/quiz2\_10.pg

If 
$$v_1 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$
 and  $v_2 = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$ 

are eigenvectors of a matrix A corresponding to the eigenvalues  $\lambda_1 = 1$  and  $\lambda_2 = -2$ , respectively, then

a. 
$$A(v_1 + v_2) =$$

- 10 • D. 6
- 12 • E. 4
- 0
- G. None of those above

b. 
$$A(-3v_1) =$$

- -6

- H. None of those above

### 133. (1 pt) local/Library/UI/Fall14/quiz2\_11.pg

Let 
$$v_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$
,  $v_2 = \begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$ , and  $v_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ 

be eigenvectors of the matrix A which correspond to the eigenvalues  $\lambda_1 = -3$ ,  $\lambda_2 = 1$ , and  $\lambda_3 = 2$ , respectively, and let

$$v = \begin{bmatrix} -5 \\ -1 \\ 0 \end{bmatrix}.$$

Express v as a linear combination of  $v_1$ ,  $v_2$ , and  $v_3$ , and find Av.

1. If 
$$v = c_1v_1 + c_2v_2 + c_3v_3$$
, then  $(c_1, c_2, c_3) =$ 

- A. (1,2,2)
- B. (-3,2,4)
- C. (-4,7,3)
- D. (-2,-1,-2)
- E. (0,1,2)
- F. (4,-1,5)
- G. None of above

#### 2. Av =

- -12
- -2 • B. 12
- -6
- 10 • D. 6
- E. 10
- 12 8 • F.
- -3 • G. 12
- H. None of those above

Suppose u and v are eigenvectors of A with eigenvalue 1 and w is an eigenvector of A with eigenvalue 2. Determine which of the following are eigenvectors of A and their corresponding eigenvalues.

- (a.) If 4v an eigenvector of A, determine its eigenvalue. Else state it is not an eigenvector of A.
  - A. -4
  - B. -3
  - C. -2
  - D. -1
  - E. 0
  - F. 1
  - G. 2
  - H. 3
  - I. 4
  - J. 4v need not be an eigenvector of A
- (b.) If 2u + 3v an eigenvector of A, determine its eigenvalue. Else state it is not an eigenvector of A.

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- A. -4
- B. -3
- C. -2
- D. -1
- E. 0F. 1
- G. 2
- U. 2
- H. 3I. 4
- J. 2u + 3v need not be an eigenvector of A
- (c.) If 2u + 3w an eigenvector of A, determine its eigenvalue. Else state it is not an eigenvector of A.
  - A. -4
  - B. -3
  - C. -2
  - D. -1
  - E. 0
  - F. 1
  - G. 2
  - H. 3
  - I. 4
  - J. 2u + 3w need not be an eigenvector of A